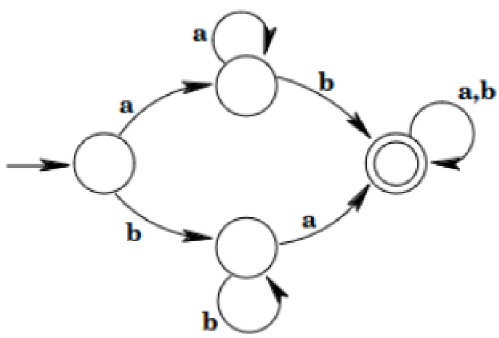
What language does the following DFA accept?

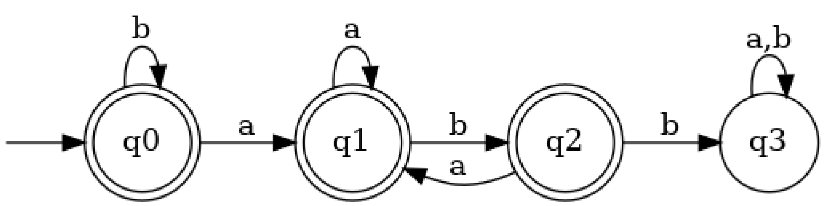


Answer: $L(M) = L((aa^*b|bb^*a)(a|b)^*) = \{x \in \{a,b\}^* \mid x \text{ contains } ab \text{ or } ba\}$

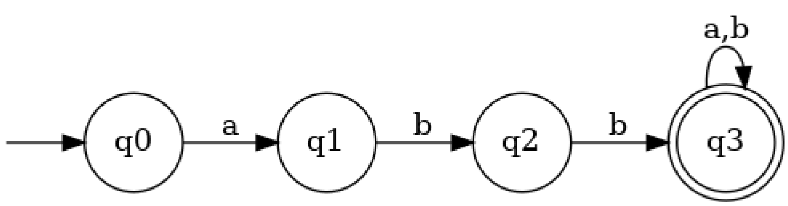
Discussion: There are two routes from the start state to the accepting state — the upper path matches strings of the form aa^*b and the lower path matches strings of the form bb^*a. Once in the accepting state, any number of as and bs can follow.

Give a DFA that accepts the language $\{x \mid x \text{ does not contain substring } abb\}$.

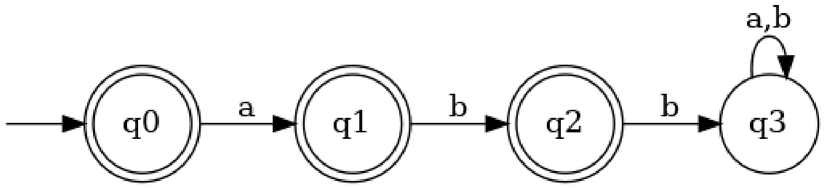
Answer:



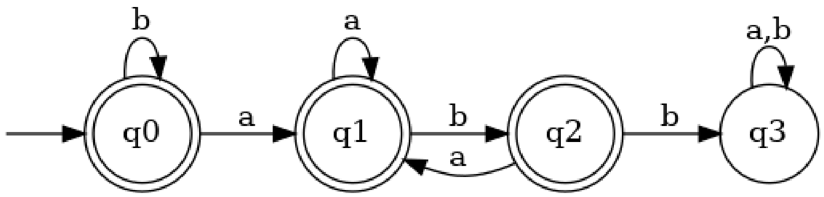
Discussion: If the task was to accept strings containing abb, we'd start with the following:



However, since only strings *not* containing abb are accepted, q_3 must be a trap state instead of an accepting state and the other states should be accepting states.

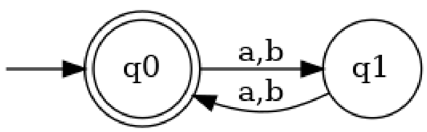


This isn't yet a complete DFA since there are missing transitions that don't necessary go to trap states. As those are considered, it is useful to keep in mind what each of the states represent: q_0 is "no part of *abb* has been seen", q_1 is "the last thing seen is *a*", q_2 is "the last thing seen is *ab*" and q_3 is "have seen *abb*".



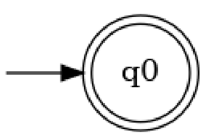
Give a DFA that accepts the language $\{x \mid n_a(x) + n_b(x) \text{ is even}\}$.

Answer:

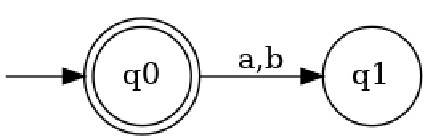


Discussion: First notice that since the alphabet contains only a and b, $n_a(x) + n_b(x)$ is even means that $|x|$ is even.

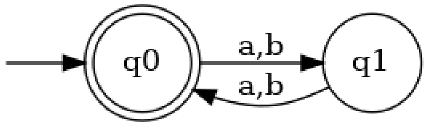
So, we need a start state. And since 0 is an even length, ϵ should be accepted and thus q_0 should be an accepting state.



Now, what happens with the next symbol? Whether a or b, a single symbol means that $|x|$ is odd.

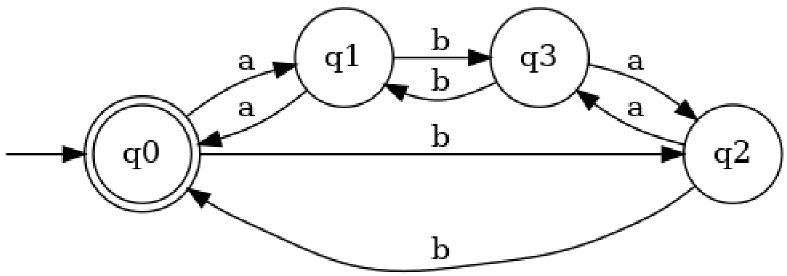


Then, another single symbol in state q_1, where $|x|$ is odd, means that $|x|$ is even.



Give a DFA that accepts the language $\{x \mid n_a(x) \text{ is even and } n_b(x) \text{ is even}\}$.

Answer:



Discussion: In the previous example, the two possibilities for $|x|$ were that $|x|$ is even and $|x|$ is odd, which corresponds to the two states in the DFA. In this case, there are four possibilities — let q_0 represent $n_a(x)$ even and $n_b(x)$ even, q_1 represent $n_a(x)$ odd and $n_b(x)$ even, q_2 represent $n_a(x)$ even and $n_b(x)$ odd, q_3 represent $n_a(x)$ odd and $n_b(x)$ odd. Then fill in the transitions from each state to the correct next state.