

Corrections —

The original version said  $n$  not divisible by 3 meant that  $n = 2k + 1$  or  $n = 2k + 2$ .

An integer  $n$  is **divisible by**  $m$  iff  $n = mk$  for some integer  $k$ . (This can also be expressed by saying that  $m$  evenly divides  $n$ .) So for example,  $n$  is divisible by 2 iff  $n = 2k$  for some integer  $k$ ;  $n$  is divisible by 3 iff  $n = 3k$  for some integer  $k$ , and so on. Note that if  $n$  is *not* divisible by 2, then  $n$  must be 1 more than a multiple of 2 so  $n = 2k + 1$  for some integer  $k$ . Similarly, if  $n$  is not divisible by 3 then  $n$  must be 1 or 2 more than a multiple of 3, so  $n = 3k + 1$  or  $n = 3k + 2$  for some integer  $k$ .

The original version incorrectly listed 10, 11, 12, etc as hexadecimal values instead of A, B, C, etc.

hexadecimal	binary	hexadecimal	binary
0	0000	8	1000
1	0001	9	1001
2	0010	A	1010
3	0011	B	1011
4	0100	C	1100
5	0101	D	1101
6	0110	E	1110
7	0111	F	1111

A  $\neg$  was left out in the third premise.

5. [12 points] Give a formal proof that the following argument is valid. Be sure to state a reason for each step in the proof. (If the rule you are using isn't named, write "unnamed rule".)

$$A \rightarrow C$$

$$C \rightarrow B$$

$$\neg B \wedge D$$

$$E \vee A$$

$$E \wedge F \rightarrow G$$

$$F$$

$$\therefore G$$

While not strictly necessary, adding parens around the  $\forall y F(y, x)$  in part (d) adds clarity.

6. [12 points] Consider the following propositions, where the domain of discourse in all cases is the set of people:

$S(x)$  stands for “ $x$  is successful”

$K(x)$  stands for “ $x$  is kind”

$F(x, y)$  stands for “ $x$  is friends with  $y$ ”

- (d) Express the proposition  $\neg \exists x ((\forall y F(y, x)) \rightarrow S(x))$  as a sentence in natural English.