The commutative law allows changing the order of operands for $\cup$ and $\cap$ ($A \cup B \equiv B \cup A$ and $A \cap B \equiv B \cap A$), but it is a rule application and thus needs to be cited as a justification (and be its own step). No hidden commutative steps!

For example, $(A \cup B) \cap \overline{B} \equiv (\overline{B} \cap A) \cup (\overline{B} \cap B)$ but this takes two steps to show:

$$\begin{aligned} (A \cup B) \cap \overline{B} &\equiv \overline{B} \cap (A \cup B) && \text{commutative law} \\ &\equiv (\overline{B} \cap A) \cup (\overline{B} \cap B) && \text{DeMorgan's law} \end{aligned}$$

The associative law only applies across the same operations — $A \cup (B \cap C) \not\equiv (A \cup B) \cap C$. (To understand this, note that everything in A is in $A \cup (B \cap C)$ but only those things in A which are also in C are in $(A \cup B) \cap C$).) As a result, use parens when there's a mix of $\cup$ and $\cap$ as the precedence rules are not clear in this situation.

By this token, #1e should have parens. Both interpretations $\overline{(A \cup B) \cap C}$ and $\overline{A \cup (B \cap C)}$ were accepted as correct.

Similarly, $(A \cup B) \cup (A \cap B) \not\equiv A \cup (B \cup A) \cap B$. (The associative law for $\cup$ has the form $(A \cup B) \cup C \equiv A \cup (B \cup C)$, so $(A \cup B) \cup (A \cap B) \equiv A \cup (B \cup (A \cap B))$.)

The basic statement of DeMorgan's Law applies to two operands, though the book showed that it can be extended to multiple instances of the same operator so you can write $\overline{A \cup B \cup C} \equiv \overline{A} \cap \overline{B} \cap \overline{C}$ with the justification "DeMorgan's Law". However, the book did not establish that it also applies across a mix of $\cup$ and $\cap$ so you need to show that chain of logical equivalences:

$$\begin{aligned} \overline{(A \cup B) \cap C} &\equiv \overline{(A \cup B)} \cup \overline{C} && \text{DeMorgan's Law} \\ &\equiv (\overline{A} \cap \overline{B}) \cup \overline{C} && \text{DeMorgan's Law} \end{aligned}$$

A similar path shows that $\overline{A \cup (B \cap C)} \equiv \overline{A} \cap (\overline{B} \cup \overline{C})$.

Double complement requires that the two complement operations be applied to the same thing. It does not directly apply to $\overline{A \cap \overline{B \cap \overline{C}}}$ — instead, first use DeMorgan's Law to get $\overline{A} \cup \overline{\overline{B \cap \overline{C}}}$ and then apply double complement to get $\overline{A} \cup (B \cap \overline{C})$.

#2 states that a, b, and c are values of type `int` in Java, which means that each have 32 bits. If not all 32 bits are written, fill in the leftmost bits with 0s. That means that c can also be written as `0x0000FFFF`.

Line up operands for bitwise operations starting from the rightmost bit (and remember to fill in the leftmost bits with 0s if they aren't specified).

```
  0x5678EF00
& 0x    FFFF
------------
  0x0000EF00
```

The 32 bit size also means that left-shifting (`<<`) loses bits from the left end.

$$\texttt{a} = \texttt{1010 1011 1100 1101 0001 0010 0011 0100}$$

so

$$(\texttt{a} << \texttt{16}) = \texttt{0001 0010 0011 0100 0000 0000 0000 0000}$$

There was an error in #4, and the question as originally written was invalid because it called for `n % i` when $i = 0$, which is a divide-by-zero error. In the corrected version, where the condition is `( n & 1 ) == 1`, the function counts the number of 1s in the binary representation of n — `( n & 1 ) == 1` if and only if the rightmost bit in n is 1, and `n = n >>> 1` shifts the next bit into the rightmost position.

Because of the error, this question was not graded and full credit was given for any attempt made.

#6 asked about both one-to-one and onto; your answer should address both and explain why or why not for each.

Make sure you explain why (or why not). "There's no possibility of having the same output for different inputs" is a only claim, and is pretty much just the definition of one-to-one. Why is there no possibility of having the same output for different inputs?

Just showing $f(1)$, $f(2)$, $f(3)$, etc for several values of n is not a proof of onto or one-to-one. (A single value can serve as a counterexample to show that something is false, but to show that something is true one must show it for all values.) Writing out some examples can be helpful for figuring out whether you are trying to show that the function is or isn't, but there either needs to be enough values written out for there to be a clear pattern visible or (better!) a general argument made that applies to any number.

Reference the definitions of "onto" and "one-to-one" given in class — for onto, this means showing that the image equals the range, and for one-to-one, this means either showing that $x \neq y \rightarrow f(x) \neq f(y)$ or that $f(x) = f(y) \rightarrow x = y$. Some people cited the horizontal line test (not discussed in class) as evidence — it's fine to bring in other knowledge you might have, but you still need to explain why that shows the appropriate definition is satisfied.

Remember that 0 is a natural number.

For #6b and #6c, an effective strategy for showing one-to-one (or not) is to consider even and odd cases separately. For example, let x and y be even. Then, for #6b, $f(x) = x + 1$ and $f(y) = y + 1$. $x + 1 = y + 1$ requires $x = y$. A similar argument can be made when x and y are both odd. But you also need to address the case when one of x and y is even and the other is odd. Together those three cases cover all the possibilities for $x, y \in \mathbb{N}$.

For onto, some people said the function in #6b wasn't onto because $f(x)$ is always odd when x is even and even when x is odd. That is true, but it isn't immediately a problem for f being onto — onto requires that for all $b \in \mathbb{N}$, there is some $a \in \mathbb{N}$ such that $f(a) = b$. Both even and odd bs need to be covered, but $f(x)$ isn't limited to only even or only odd numbers.

(To show that #6b is onto: observe that if a is even, then $b = a + 1$ and b is odd; if a is odd, then $b = a - 1$ and b is even. Both even and odd bs are covered here, so we just need to make sure that there aren't gaps and that $a \in \mathbb{N}$ for all $b \in \mathbb{N}$. Check for gaps by first considering $f(0)$ and $f(1)$, the smallest $a \in \mathbb{N}$: $f(0) = 1$, $f(1) = 0$. No bs skipped at the beginning. The consider successive even as and odd as — if a is even, $f(a) = a + 1$ and $f(a + 2) = a + 2 + 1 = a + 3$. No odd bs skipped here. It is similar for odd as — $f(a) = a - 1$ and $f(a + 2) = a + 2 - 1 = a + 1$. No even bs skipped. The last step is to make sure $a \in \mathbb{N}$. With $b = a + 1$ or $b = a - 1$, the only way $a \notin \mathbb{N}$ when $b \in \mathbb{N}$ is if $b = a - 1$ applies when $b = 0$. But $a = b - 1$ is the case when b is odd, not even, and 0 is even.)

For #6c, you don't need to consider the cases of $f(x)$ being even and $f(x)$ being odd separately in order to show that f is onto. First, the even/odd distinction in the definition of f applies to x, not $f(x)$. But also, consider $f(a) = b$. Choose $a = 2b$. $f(2b) = 2b/2 = b$ because $2b$ is always even, so this satisfies showing that for each $b \in \mathbb{N}$, there is an $a \in \mathbb{N}$ such that $f(a) = b$.