Write strings generated by the following grammar. Illustrate the different possibilities.

$$
\begin{aligned}
E &::= T\ [\ +\ T\ ]\ldots \\
T &::= F\ [\ *\ F\ ]\ldots \\
F &::= \text{"("}\ E\ \text{")"}\ \mid\ x\ \mid\ y\ \mid\ z
\end{aligned}
$$

Answer: x, $(x + y * x)$, $((x * y) + x * z)$ — these show the different operators and how expressions can be nested.

Discussion: "Illustrate the different possibilities" means to cover the different rules to show the various options. Some examples:

$$
\begin{aligned}
E &\Longrightarrow T \\
&\Longrightarrow F \\
&\Longrightarrow x
\end{aligned}
$$

$$
\begin{aligned}
E &\Longrightarrow T \\
&\Longrightarrow F \\
&\Longrightarrow \text{"("}\ E\ \text{")"} \\
&\Longrightarrow \text{"("}\ T\ +\ T\ \text{")"} \\
&\Longrightarrow \text{"("}\ F\ +\ T\ \text{")"} \\
&\Longrightarrow \text{"("}\ F\ +\ F\ *\ F\ \text{")"} \\
&\Longrightarrow \text{"("}\ x\ +\ F\ *\ F\ \text{")"} \\
&\Longrightarrow \text{"("}\ x\ +\ y\ *\ F\ \text{")"} \\
&\Longrightarrow \text{"("}\ x\ +\ y\ *\ z\ \text{")"}
\end{aligned}
$$

$$
\begin{aligned}
E &\Longrightarrow T \\
&\Longrightarrow F \\
&\Longrightarrow \text{"("}\ E\ \text{")"} \\
&\Longrightarrow \text{"("}\ T\ +\ T\ \text{")"} \\
&\Longrightarrow \text{"("}\ F\ +\ T\ \text{")"} \\
&\Longrightarrow \text{"("}\ \text{"("}\ E\ \text{")"}\ +\ T\ \text{")"} \\
&\Longrightarrow \text{"("}\ \text{"("}\ T\ \text{")"}\ +\ T\ \text{")"} \\
&\Longrightarrow \text{"("}\ \text{"("}\ F\ *\ F\ \text{")"}\ +\ T\ \text{")"} \\
&\Longrightarrow \text{"("}\ \text{"("}\ x\ *\ F\ \text{")"}\ +\ T\ \text{")"} \\
&\Longrightarrow \text{"("}\ \text{"("}\ x\ *\ y\ \text{")"}\ +\ T\ \text{")"} \\
&\Longrightarrow \text{"("}\ \text{"("}\ x\ *\ y\ \text{")"}\ +\ F\ *\ F\ \text{")"} \\
&\Longrightarrow \text{"("}\ \text{"("}\ x\ *\ y\ \text{")"}\ +\ x\ *\ F\ \text{")"} \\
&\Longrightarrow \text{"("}\ \text{"("}\ x\ *\ y\ \text{")"}\ +\ x\ *\ z\ \text{")"}
\end{aligned}
$$

Write the following BNF grammar using the standard context-free grammar notation.

$$
\begin{aligned}
&E ::= T\ [\ +\ T\ ] \ldots \\
&T ::= F\ [\ *\ F\ ] \ldots \\
&F ::= \text{"("}\ E\ \text{")"}\ |\ x\ |\ y\ |\ z
\end{aligned}
$$

Answer:

$$
\begin{aligned}
E &\longrightarrow TA \\
A &\longrightarrow +TA \\
A &\longrightarrow \epsilon \\
T &\longrightarrow FB \\
B &\longrightarrow +FB \\
B &\longrightarrow \epsilon \\
F &\longrightarrow \text{"("}\ E\ \text{")"} \\
F &\longrightarrow x \\
F &\longrightarrow y \\
F &\longrightarrow z
\end{aligned}
$$

Discussion: The rule $F ::= \text{"("}\ E\ \text{")"}\ |\ x\ |\ y\ |\ z$ expresses four alternatives for F:

$$
\begin{aligned}
F &\longrightarrow \text{"("}\ E\ \text{")"} \\
F &\longrightarrow x \\
F &\longrightarrow y \\
F &\longrightarrow z
\end{aligned}
$$

For $E ::= T\ [\ +\ T\ ] \ldots$, there are two possibilities — $E \longrightarrow T$ and something which can produce repetitions of $+\ T$ following a T. Repetitions takes the form $A \longrightarrow +TA$ — a new non-terminal is needed besides E because of the initial T in $T\ [\ +\ T\ ] \ldots$. $A \longrightarrow \epsilon$ ends the repetition. Finally, the "following a T part" comes from a rule $E \longrightarrow TA$.

$$
\begin{aligned}
E &\longrightarrow TA \\
A &\longrightarrow +TA \\
A &\longrightarrow \epsilon
\end{aligned}
$$

($E \longrightarrow T$ isn't needed because $E \Longrightarrow T$ can be derived from $E \longrightarrow TA$ and $A \longrightarrow \epsilon$.)

The same strategy can be used for the final BNF rule:

$$T \longrightarrow FB$$
$$B \longrightarrow +FB$$
$$B \longrightarrow \epsilon$$

Show that the following grammar is ambiguous by finding a string that has two left derivations according to the grammar.

$$\begin{aligned} S &\longrightarrow SS \\ S &\longrightarrow aSb \\ S &\longrightarrow bSa \\ S &\longrightarrow \epsilon \end{aligned}$$

Answer: $aabb$ is a string with two left derivations:

$$\begin{aligned} S &\Longrightarrow aSb \\ S &\Longrightarrow aaSbb \\ S &\Longrightarrow aabb \end{aligned} \qquad\qquad \begin{aligned} S &\Longrightarrow SS \\ &\Longrightarrow aSbS \\ &\Longrightarrow aaSbbS \\ &\Longrightarrow aabbS \\ &\Longrightarrow aabb \end{aligned}$$

Discussion: The goal is to find two left derivations that lead to the same string, so a strategy is to start off with applying different rules — thus the derivations will be different — and then try to get both derivations to the same string.

Start each derivation with a different rule:

$$S \Longrightarrow aSb \qquad\qquad S \Longrightarrow bSa$$

But we can see that in the first one, whatever string is derived will start with a and end with b, while the opposite is true in the second derivation. These derivations will never result in the same string.

Try something else —

$$S \Longrightarrow aSb \qquad\qquad S \Longrightarrow SS$$

Since the first derivation will result in a string starting with a and ending with b, we need to aim for that in the second derivation as well.

$$S \Longrightarrow aSb \qquad\qquad \begin{aligned} S &\Longrightarrow SS \\ &\Longrightarrow aSbS \end{aligned}$$

Now apply the same steps to each derivation.

$$
\begin{aligned}
S &\Longrightarrow aSb & \qquad S &\Longrightarrow SS \\
S &\Longrightarrow aaSbb & &\Longrightarrow aSbS \\
S &\Longrightarrow aabb & &\Longrightarrow aaSbbS \\
& & &\Longrightarrow aabbS
\end{aligned}
$$

Finally, eliminate the final S on the right side.

$$
\begin{aligned}
S &\Longrightarrow aSb & \qquad S &\Longrightarrow SS \\
S &\Longrightarrow aaSbb & &\Longrightarrow aSbS \\
S &\Longrightarrow aabb & &\Longrightarrow aaSbbS \\
& & &\Longrightarrow aabbS \\
& & &\Longrightarrow aabb
\end{aligned}
$$

Find a left derivation for $(x + y) * z$ in the following grammar.

$$
\begin{aligned}
E &\longrightarrow E \ + \ T \\
E &\longrightarrow T \\
T &\longrightarrow T \ * \ F \\
T &\longrightarrow F \\
F &\longrightarrow (E) \\
F &\longrightarrow x \\
F &\longrightarrow y \\
F &\longrightarrow z
\end{aligned}
$$

Answer:

$$
\begin{aligned}
E &\Longrightarrow T \\
&\Longrightarrow T * F \\
&\Longrightarrow F * F \\
&\Longrightarrow (E) * F \\
&\Longrightarrow (E + T) * F \\
&\Longrightarrow (T + T) * F \\
&\Longrightarrow (F + T) * F \\
&\Longrightarrow (x + T) * F \\
&\Longrightarrow (x + F) * F \\
&\Longrightarrow (x + y) * F \\
&\Longrightarrow (x + y) * z
\end{aligned}
$$

Discussion: What is interesting here is not the derivation itself, but the process — the first decision, for example, is between $E \longrightarrow E + T$ and $\longrightarrow T$. Which to choose? Just looking at the first symbol $(x + y) * z$ isn't enough.