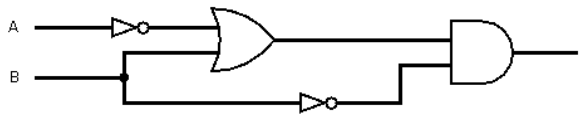
This homework covers sections 1.3-1.4. It is due in class Friday, February 6. Hand in a hardcopy of your solutions.

When writing your solutions, write your answers carefully, giving attention to clearly presenting the solution, and be sure to always show your work and explain your reasoning. Unsupported answers will receive little or no credit.

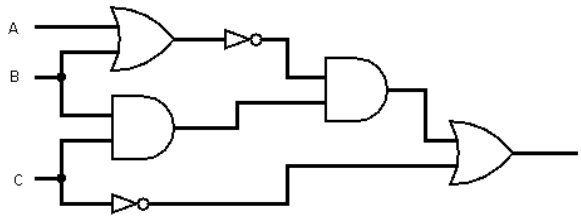
While you may discuss ideas and strategies for problems with other students, you should always make the first attempt on a problem yourself and **you must write up your own solutions in your own words**. *You may not collaboratively write solutions or copy a solution that one person in the group writes up. You also may not look for, copy, or use solutions from other sources, including from generative AI like ChatGPT, even if you make changes. There's no such thing as using someone else's solution for a problem "as an example" for writing your own.*

1. Find the proposition that gives the output of each circuit as a function of its inputs. Show your work by redrawing the circuit and labeling the output of each logic gate.

(a)



(b)



2. Draw a logic circuit that computes each of the following propositions. Do not simplify the proposition first; build the circuit to directly correspond to what is written.

   (a) $(A \wedge \neg B) \vee (B \wedge C)$

   (b) $(A \vee \neg B \vee C) \wedge \neg(A \wedge \neg B \wedge C)$

3. Simplify each of the following expressions. In particular, $\neg$ should only be applied to individual predicates. Show your work by writing a chain of logical equivalences starting from the given expression. Provide a justification for each step by identifying the rule or definition used.

   (a) $\neg((\forall x P(x)) \vee (\forall x Q(x)))$

   (b) $\neg\forall s((\exists t L(s,t)) \vee (\exists t L(t,s)))$

(c) $\neg\forall n(Z(n) \rightarrow \exists k(Z(k) \wedge G(k, n)))$

(d) $\neg\exists x\exists y(R(x, y) \wedge \forall z(S(x, z) \vee T(y, z)))$

4. The following boolean expressions are not logically equivalent. Explain the difference in meaning, and give an example of two specific predicates P and Q for which one expression is true and the other is false. (For example, for P and Q, you might consider predicates involving properties of integers.)

$$\exists x P(x)) \wedge (\exists x Q(x))$$

$$\exists x(P(x) \wedge Q(x))$$

5. Express the following statements in predicate logic. Give the meaning of each predicate that you use and, where it isn't clear, state the domain of discourse.

   Try to express as much of the meaning of each sentence as possible. As part of this, you should avoid overly limiting (and thus needing to state) the domain of discourse. See the discussion about "roses are red" on page 33 in the text for an example.

   If there's anything in the statement that you can't capture, explain.

   (a) It's either fun at the time or makes a good story later.

   (b) Not all that glitters is gold.

   (c) Everyone who owns a black cat is unlucky.

   (d) There is a simple solution for this problem.

   (e) Every problem has a simple solution.

   (f) Some problems have more than one solution.

   (g) There is a book that no one has read.

   (h) In a perfect world, every dog would have a home and every home would have a dog.