This homework covers sections 1.5–1.7. It is due in class Friday, February 13. Hand in a hardcopy of your solutions.

Questions 1–3 are about formal proofs of validity, as in section 1.5. For questions 4–9, you should give informal, but careful and complete, proofs as in sections 1.6 and 1.7. You can use the definitions and facts on pages 52–53 in the book, basic facts from algebra, and that the number $\sqrt{2}$ is irrational without having to prove them.

While you may discuss ideas and strategies for problems with other students, you should always make the first attempt on a problem yourself and **you must write up your own solutions in your own words**. *You may not collaboratively write solutions or copy a solution that one person in the group writes up. You also may not look for, copy, or use solutions from other sources, including from generative AI like ChatGPT, even if you make changes. There's no such thing as using someone else's solution for a problem "as an example" for writing your own.*

1. Use a truth table to show that the following argument is valid. Also explain in English why this makes sense — what does the argument mean?

$$\begin{array}{l} p \to q \\ \neg p \to q \\ \hline \therefore q \end{array}$$

2. Give a formal proof for each of the following valid arguments. Give a justification for each step in your proofs.

a) $$\begin{array}{l} (p \wedge \neg q) \to r \\ q \to \neg p \\ p \\ \hline \therefore r \end{array}$$

b) $$\begin{array}{l} (r \wedge s) \to p \\ \neg(p \wedge q) \\ r \\ q \\ \hline \therefore \neg s \end{array}$$

c) $$\begin{array}{l} p \to r \\ (r \wedge s) \to t \\ q \to \neg t \\ s \\ q \\ \hline \therefore \neg p \end{array}$$

3. Translate each of the following arguments into formal logic and determine whether the argument is valid. If the argument is valid, give a formal proof. If the argument is not valid, explain why.

   (a) If this card is red, then it's the king of hearts. But if this card is a king, then it is not a heart. So, this card is not red.

   (b) A math major takes Abstract Algebra or Foundations of Analysis. If someone takes Abstract Algebra, then that person knows about Galois. Alice graduated with a major in math, but she never took Foundations of Analysis. So, Alice knows about Galois.

(c) If Bob fails this course, then he won't graduate on time. If Bob doesn't study, then he will fail this course. Bob studies. So Bob will graduate on time.

(d) If Penelope is at the party, then so is Quincy. If Quincy and Rachel are both at the party, then so is Simon. Simon is not at the party, but Rachel is. So, Penelope is not at the party.

4. Prove or disprove:

   (a) If n is an integer and n is divisible by 4, then n^2 is divisible by 4.

   (b) If n is an integer and n^2 is divisible by 4, then n is divisible by 4.

5. Prove that for any integers a, b, and c, if b is divisible by a, then bc is divisible by a.

6. Prove that for any integer n, the number n^2+n is even. Hint: use proof by cases, showing separately that $n^2 + n$ is even for any even integer n and for any odd integer n.

7. Prove that an integer is divisible by 3 iff n^2 is divisible by 3. Hint: use an indirect proof in one direction.

8. (a) Prove that for any real number x, if x^2 is an irrational number, then x is also irrational. Hint: use an indirect proof.

   (b) Disprove that for any real number x, if x is an irrational number, then x^2 is also irrational.

9. Prove that if x is an irrational number and r is a non-zero rational number, xr is an irrational number. Hint: use proof by contradiction.