This homework covers sections 3.1–3.2 and 3.4. It is due in class Friday, March 27. Hand in a hardcopy of your solutions.

While you may discuss problems with other students, you should always make the first attempt on a problem yourself and **you must write up your own solutions in your own words**. *You may not collaboratively write solutions or copy a solution that one person in the group writes up.*

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1. Let L and M be languages over the alphabet $\Sigma = \{a, b\}$ where

$$L = \{a, aa\} \qquad M = \{x \in \Sigma^* \mid x \text{ ends with } b\}$$

Find the following languages — write the language out as a set or give a clear English description of the language. You do not need to justify your answers, but an explanation can help you get partial credit for an incorrect answer.

a) $L \cap M$ b) $L \cup M$ c) L^3 d) L^* e) M^*

f) ML g) LM h) $\overline{M}$ i) M^R j) $M^R M$

2. For each of the following, give a clear, concise, simple English description of the language generated by the regular expression over Σ. (Simply describing the regular expression — e.g. any number of as, then a b, then any number of as — is not an acceptable answer. This language would be better described as "strings with exactly one b".) Hint: it can help to start writing out, in some methodical way, strings matching the pattern.

(a) Let $\Sigma = \{a, b\}$.

(i) bab^* (ii) $b(ab)^*$ (iii) $(a|b)^*bb(a|b)$

(iv) $a^*(b|\epsilon)a^*(b|\epsilon)a^*(b|\epsilon)a^*$

(b) Let $\Sigma = \{a, b, c\}$.

(i) $ab(a|b|c)^*ba \mid aba$ (ii) $((a|\epsilon)(b|c))^*(a|\epsilon)$

3. For each of the following languages, give a regular expression that generates the language. Justify your answers by explaining why the regular expression generates the strings of the language. Be careful to note the alphabet in each case, and be careful to account for all of the strings that satisfy the given condition.

   (a) $\{x \in \{a,b\}^* \mid |x| \geq 2$ and x starts and ends with the same symbol$\}$

   (b) $\{x \in \{a,b\}^* \mid x$ contains at least one a and at least one $b\}$

   (c) $\{x \in \{a,b,c\}^* \mid |x|$ is a multiple of 3$\}$

   (d) $\{x \in \{a,b,c\}^* \mid x$ doesn't start with a $b\}$

   (e) $\{x \in \{a,b,c\}^* \mid$ every a in x is immediately followed by a $b\}$

   (f) $\{x \in \{a,b,c\}^* \mid x$ contains both ab and $ba\}$

4. Let DFA $M = \{Q, \Sigma, p_1, \delta, F\}$ where

$$Q = \{p_1, p_2, p_3, p_4\}, \qquad \Sigma = \{a,b,c\}, \qquad F = \{p_2, p_4\},$$

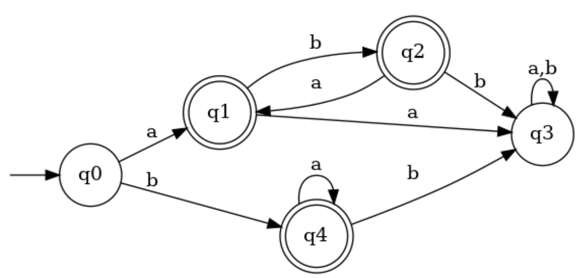
   and δ is given by the table below.

	p_1	p_2	p_3	p_4
a	p_2	p_2	p_3	p_3
b	p_4	p_3	p_3	p_4
c	p_1	p_3	p_3	p_4

   (a) Draw a transition diagram for M.

   (b) Based on the diagram, find a regular expression for the language that is accepted by M. Explain your reasoning.

5. Consider the DFA M defined by the transition diagram shown below.



   (a) Let $M = \{Q, \Sigma, q_0, \delta, F\}$. Identify Q, Σ, δ, and F. For δ, give the transition table.

   (b) Find a regular expression for the language that is accepted by M. Explain your reasoning.

6. For each of the following languages, draw a transition diagram for a DFA that accepts that language, that is, it accepts all the strings in the language and no other strings. Note the alphabet in each case — the alphabet for the DFA should be the same as the alphabet for the language.

   (a) $\{w \in \{a, b\}^* \mid w \text{ ends with the string } abab\}$

   (b) $\{w \in \{a, b, c\}^* \mid w \text{ contains a } c \text{ and there are no } a\text{s after the first } c\}$

   (c) $\{w \in \{a, b, c\}^* \mid n_a(w) + n_b(w) \text{ is a multiple of } 3\}$