This homework covers sections 3.7 and 4.1, with a little bit left over from section 3.6. It is due in class Friday, April 10. Hand in a hardcopy of your solutions.

While you may discuss problems with other students, you should always make the first attempt on a problem yourself and **you must write up your own solutions in your own words**. *You may not collaboratively write solutions or copy a solution that one person in the group writes up.*

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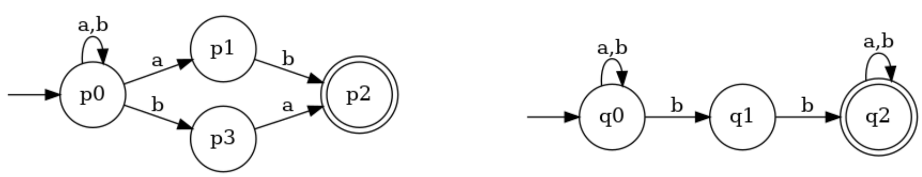
1. Use the construction outlined in the proof of Theorem 3.5 to construct a machine that accepts the intersection of the languages accepted by the two NFAs shown. (Hint: remember that the construction outlined in the proof of Theorem 3.5 is for two *DFAs*.)



2. Use the Pumping Lemma (Theorem 3.6) to prove that the following languages are not regular.

   Remember the pattern for the proof discussed in class, which can be written:

   > Suppose L is regular. Then, by the Pumping Lemma, there is an integer N such that for any string $w \in L$ with $|w| \geq N$, $w = xyz$ where $|xy| \leq N$, $|y| \geq 1$, and $xy^nz \in L$ for all natural numbers n. But let $w =$ _____, which is in L with $|w| \geq N$. Write w as xyz. Because $|xy| \leq N$, y must be of the form _____. But then $xy^nz \notin L$ for $n =$_____ because ____________________.

   Fill in the blanks.

   (a) $L = \{ \ a^nb^nc^n \mid n \in \mathbb{N} \ \}$

   (b) $L = \{ \ w \in \{a, b\}^* \mid n_a(w) < n_b(w) \ \}$, where $n_\sigma(w)$ means the number of σs in w

3. Consider the context-free grammar shown on the right.

$$\begin{aligned} S &\longrightarrow TR \\ T &\longrightarrow aTb \\ T &\longrightarrow \epsilon \\ R &\longrightarrow cRd \\ R &\longrightarrow c \end{aligned}$$

   (a) Write a derivation for the string *aabbc* using this grammar.

   (b) Write a derivation for the string *abccccdd* using this grammar.

   (c) Find the language generated by this grammar. Briefly justify your answer.

4. For each of the following languages, give a context-free grammar that generates the language. Also explain in words why your grammar works.

   Hint for part (d): keep in mind how to generate equal numbers of things, and think about which pairs of characters can match up with each other.

   a) $\{a^n b^m \mid n \neq m\}$
   b) $\{a^n b^m c^k \mid m > n + k\}$
   c) $\{a^n b^m c^k d^l \mid m = k \text{ and } n = l\}$
   d) $\{a^n b^m c^k d^l \mid n + m = k + l\}$