

This homework covers sections 4.3–4.4. It is due in class Monday, April 20. Hand in a hardcopy of your solutions.

*While you may discuss problems with other students, you should always make the first attempt on a problem yourself and **you must write up your own solutions in your own words.** You may not collaboratively write solutions or copy a solution that one person in the group writes up.*

1. A context-free grammar for a simplified version of propositional logic is shown.

$$S \rightarrow A$$

$$S \rightarrow B$$

$$A \rightarrow p$$

$$A \rightarrow q$$

$$A \rightarrow r$$

$$A \rightarrow s$$

$$B \rightarrow \neg F$$

$$B \rightarrow F \wedge F$$

$$B \rightarrow F \vee F$$

$$B \rightarrow F \rightarrow F$$

$$F \rightarrow A$$

$$F \rightarrow (B)$$

- (a) Draw a parse tree for the proposition

$$(r \rightarrow p) \wedge q$$

using this grammar.

- (b) Draw a parse tree for the proposition

$$((\neg r) \wedge q) \rightarrow ((p \vee r) \wedge s)$$

using this grammar.

2. The context-free grammar shown generates the language

$$S \rightarrow ARS$$

$$A \rightarrow abb$$

$$R \rightarrow bR$$

$$R \rightarrow cR$$

$$S \rightarrow \epsilon$$

$$A \rightarrow \epsilon$$

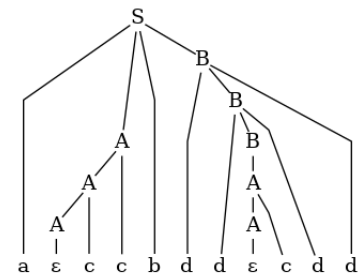
$$R \rightarrow \epsilon$$

$$\{w \in \{a, b, c\}^* \mid \text{every } a \text{ is followed by at least two } bs \}$$

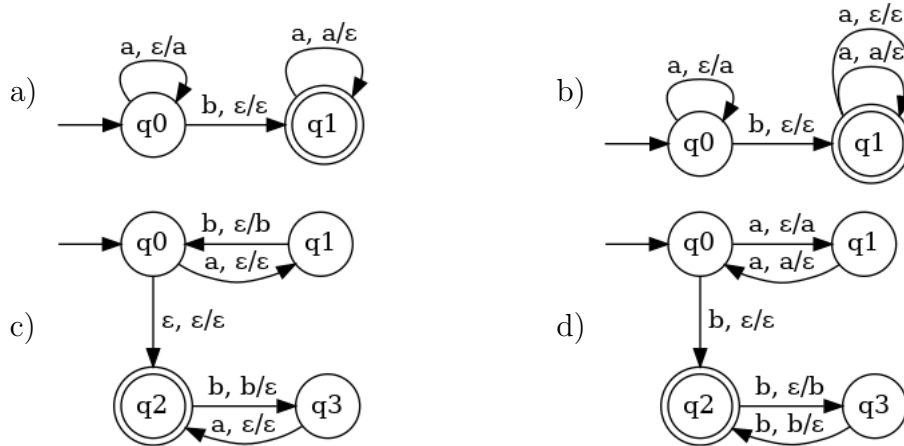
Show that this grammar is ambiguous by giving either two parse trees or two left derivations for the string *cabbabbc*.

3. Consider the parse tree shown. It is based on a context-free grammar *G*.

- (a) Give the production rules that must be part of *G* in order for the parse tree to be valid.
- (b) What is the string being parsed by this parse tree?
- (c) Give the left derivation corresponding to this parse tree.
- (d) Give the right derivation corresponding to this parse tree.



4. Give a concise description of the language accepted by each of the following pushdown automata. Explain your answer by describing how the machine works when accepting a string in the language.



5. For each of the following, draw a transition diagram for a pushdown automaton that accepts the language. Explain how your machines work.

(a) $L = \{ a^n b^n \mid n \text{ is a multiple of } 3 \}$

- (b) Let L be the language over the alphabet $\{(,), [,], a, b\}$ that consists of strings where the parentheses and brackets are properly nested. That is, each opening (or [is followed by a corresponding) or], each closing) or] is preceded by a corresponding (or], and within each matching pair, any other (, [,), and] symbols are also properly nested. For example, $(ab())[bb]$ is properly nested but $(a[b])a$, $([ba][aba)$, and ba are not.

6. Show that the language $L = \{ w \in \{a, b\}^* \mid n_a(w) > n_b(w) \}$ is deterministic context-free.