## Key Points

- terms and concepts: premise, conclusion, logically deduced, argument, valid argument, formal proof
- important rules of deduction
  - *modus ponens* (both propositional logic and predicate logic forms)
  - *modus tollens* (both propositional logic and predicate logic forms)
  - Law of Syllogism
- processes
  - proving conclusions through truth tables
  - proving conclusions through a chain of logical deductions (formal proof)
  - showing that an argument is invalid
- applications to arguments stated in English



## Premises, Conclusions, Arguments, and Proof

- a *premise* is a proposition known or taken to be true
- a *conclusion* is a proposition that can be deduced logically from the premises
  - not necessarily true in any absolute sense, just true if the premises are true
- an *argument* is a claim that a particular conclusion follows logically from a given set of premises
  - a *valid argument* is one where that claim is true
- a *formal proof* that an argument is valid consists of a sequence of propositions such that
  - the last is the conclusion, and
  - every proposition is either a premise or follows by logical deduction from earlier premises in the list, and
  - each step has a justification



## Proving a Conclusion Using a Truth Table

- proving a conclusion is equivalent to saying that
  the conjunction of the premises implies the conclusion
  is a tautology

$$\begin{array}{l} p \rightarrow q \\ p \\ \hline \therefore\ q \end{array}$$

| $p$ | $q$ | $p \rightarrow q$ |
|---|---|---|
| false | false | true |
| false | true | true |
| true | false | false |
| true | true | true |

p ∧ (p→q)

(p ∧ (p→q)) →q

- $\mathcal{P} \Rightarrow \mathcal{Q}$ means that $\mathcal{P} \rightarrow \mathcal{Q}$ is a tautology for any formulas $\mathcal{P}$, $\mathcal{Q}$



**2.** Each of the following is a valid rule of deduction. For each one, give an example of a valid argument in English that uses that rule.

$$\begin{array}{l} p \vee q \\ \neg p \\ \hline \therefore\ q \end{array} \qquad \begin{array}{l} p \\ q \\ \hline \therefore\ p \wedge q \end{array} \qquad \begin{array}{l} p \wedge q \\ \hline \therefore\ p \end{array} \qquad \begin{array}{l} p \\ \hline \therefore\ p \vee q \end{array}$$

If it is either a cat or a dog, and it is not a cat, then it must be a dog.

If it is a bird and it can fly, then it is a bird.

If it is raining, then it is either raining or sunny.

If it is an animal and it is furry, then it is a furry animal.

## Invalid Arguments

- show that an argument is *invalid* by finding an assignment of truth values to the propositional variables which makes all of the premises true but the conclusion is false

**3.** There are two notorious invalid arguments that look deceptively like *modus ponens* and *modus tollens*:

$$\begin{array}{l} p \to q \\ q \\ \hline \therefore\ p \end{array} \qquad \begin{array}{l} p \to q \\ \neg p \\ \hline \therefore\ \neg q \end{array}$$

Show that each of these arguments is invalid. Give an English example that uses each of these arguments.

If it is raining, then the ground is wet. The ground is wet, therefore it is raining.

If it is raining, then the ground is wet. It is not raining, therefore the ground is not wet.



## Rules of Deduction

- *modus ponens*

$$\begin{array}{l} p \to q \\ p \\ \hline \therefore\ q \end{array} \qquad \begin{array}{l} \forall x(P(x) \to Q(x)) \\ P(a) \\ \hline \therefore\ Q(a) \end{array} \qquad \begin{array}{l} \forall x(P(x) \to Q(x)) \\ \neg Q(a) \\ \hline \therefore\ \neg P(a) \end{array}$$

- *modus tollens*

$$\begin{array}{l} p \to q \\ \neg q \\ \hline \therefore\ \neg p \end{array}$$

- Law of Syllogism

$$\begin{array}{l} p \to q \\ q \to r \\ \hline \therefore\ p \to r \end{array}$$

- other rules

$$\begin{array}{l} p \vee q \\ \neg p \\ \hline \therefore\ q \end{array} \qquad \begin{array}{l} p \\ q \\ \hline \therefore\ p \wedge q \end{array} \qquad \begin{array}{l} p \wedge q \\ \hline \therefore\ p \end{array} \qquad \begin{array}{l} p \\ \hline \therefore\ p \vee q \end{array}$$

$\mathcal{P} \equiv \mathcal{Q}$ means $\mathcal{Q} \Rightarrow \mathcal{P}$ AND $\mathcal{P} \Rightarrow \mathcal{Q}$



**4.** Decide whether each of the following arguments is valid. If it is valid, give a formal proof. If it is invalid, show that it is invalid by finding an appropriate assignment of truth values to propositional variables.

**a)** $$\begin{array}{l} p \to q \\ q \to s \\ s \\ \hline \therefore\ p \end{array}$$

**b)** $$\begin{array}{l} p \wedge q \\ q \to (r \vee s) \\ \neg r \\ \hline \therefore\ s \end{array}$$

**c)** $$\begin{array}{l} p \vee q \\ q \to (r \wedge s) \\ \neg p \\ \hline \therefore\ s \end{array}$$

**d)** $$\begin{array}{l} (\neg p) \to t \\ q \to s \\ r \to q \\ \neg(q \vee t) \\ \hline \therefore\ p \end{array}$$

**e)** $$\begin{array}{l} p \\ s \to r \\ q \vee r \\ q \to \neg p \\ \hline \therefore\ \neg s \end{array}$$

**f)** $$\begin{array}{l} q \to t \\ p \to (t \to s) \\ p \\ \hline \therefore\ q \to s \end{array}$$

**5.** For each of the following English arguments, express the argument in terms of propositional logic and determine whether the argument is valid or invalid.

**a)** If it is Sunday, it rains or snows. Today, it is Sunday and it's not raining. Therefore, it must be snowing.

**b)** If there are anchovies on the pizza, Jack won't eat it. If Jack doesn't eat pizza, he gets angry. Jack is angry. Therefore, there were anchovies on the pizza.

**c)** At 8:00, Jane studies in the library or works at home. It's 8:00 and Jane is not studying in the library. So she must be working at home.