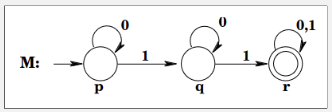
Recognizing Languages

The ***language accepted by*** M, denoted $L(M)$, is the set of all strings $w \in \Sigma^*$ that are accepted by M: $L(M) = \{w \in \Sigma^* \mid \delta^*(q_0, w) \in F\}$.

$\delta^*(q,w)$ = state after consuming w, starting from state q

$\delta^*(q,\varepsilon) = q$

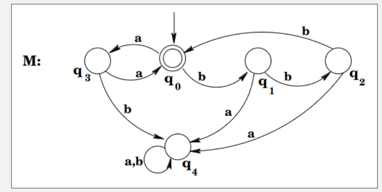
- the states act as a memory for what has been matched so far in the string
- the transitions capture what can come next
- the accepting states indicate when the match is complete



$L(M) = \{ x \in \{0,1\}^* \mid n_1(x) \geq 2 \}$

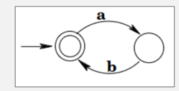
$L(M) = L(0^*10^*1(0|1)^*)$

Shortcuts



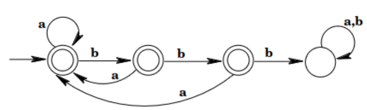
$L(M) = L((aa|bbb)^*)$

- q_4 is a garbage or trap state – a non-accepting state which is not possible to escape
  - reflects having encountered something that disqualifies the string from being in the language
- such states are commonly omitted from the transition diagram
  - this is no longer a complete DFA! (but it is understood how to turn it into a complete DFA)

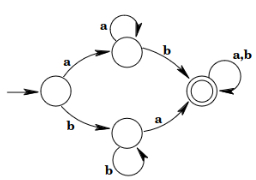


2. What languages do the following DFAs accept?

a)



b)



1. Give DFAs that accept the following languages over $\Sigma = \{a, b\}$.

a) $L_1 = \{x \mid x \text{ contains the substring } aba\}$
b) $L_2 = L(a^*b^*)$
c) $L_3 = \{x \mid n_a(x) + n_b(x) \text{ is even }\}$
d) $L_4 = \{x \mid n_a(x) \text{ is a multiple of 5 }\}$
e) $L_5 = \{x \mid x \text{ does not contain the substring } abb\}$
f) $L_6 = \{x \mid x \text{ has no } a\text{'s in the even positions}\}$
g) $L_7 = L(aa^* \mid aba^*b^*)$

3. Let $\Sigma = \{0, 1\}$. Give a DFA that accepts the language

$$L = \{x \in \Sigma^* \mid x \text{ is the binary representation of an integer divisible by 3}\}.$$