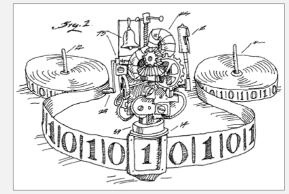
Turing Machines and Computability

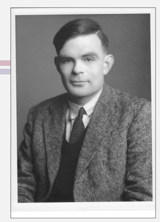
The Big Picture

- the Turing machine as a model of computation
- definitions and terminology
  - Turing machine
  - running with input *w*
  - halting with output *x*
  - Turing-computable / computes a function
  - Turing-decidable / decides a language
  - Turing-acceptable / accepts a language

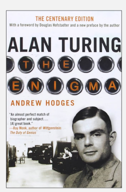


https://www.felienne.com/archives/2974

Alan Turing



- English mathematician, 1912-1954
- known for
  - fundamental contributions to theoretical computer science
    - Turing machines
    - Church-Turing thesis
  - fundamental contributions to artificial intelligence
    - Turing test
  - developments in codebreaking and crucial contributions to the cracking of Enigma (World War II)
  - contributions to mathematical biology
- the Turing Award (1966-) is the highest in computer science
- convicted of homosexual acts in 1952
  - official apology in 2009
  - royal pardon from Queen Elizabeth II in 2013
  - 2017 "Alan Turing law" retroactively pardoned others similarly convicted

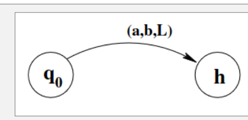


https://www.nytimes.com/2019/06/05/obituaries/alan-turing-overlooked.html
https://en.wikipedia.org/wiki/The_Imitation_Game

Turing Machines

Definition 5.1. A ***Turing machine*** is a 4-tuple $(Q, \Lambda, q_0, \delta)$, where:

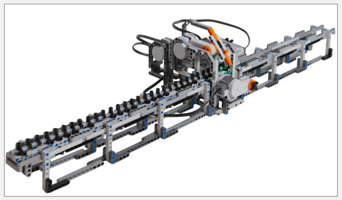
1. Q is a finite set of states, including the halt state, h.
2. Λ is an alphabet which includes the blank symbol, #.
3. $q_0 \in Q$ is the start state.
4. $\delta\colon (Q \smallsetminus \{h\}) \times \Lambda \to \Lambda \times \{L, R\} \times Q$ is the transition function. The fact that $\delta(q, \sigma) = (\tau, d, r)$ means that when the Turing machine is in state q and reads the symbol σ, it writes the symbol τ, moves one cell in the direction d, and enters state r.



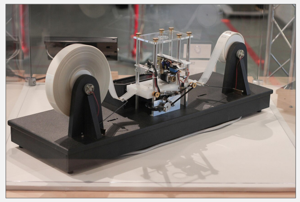
transition diagram notation – in state q_0, read *a*, write *b*, move *L*, go to state *h* (halt state)

- *running M with input w* – start with a tape that is blank except for *w*, the machine placed on the first cell of *w*, and the machine in state q_0
- *M halts with output x* – end with a tape that is blank except for *x*, the machine placed on the first cell of *x*, and the machine in state *h*
- other outcomes – *M* fails to halt, *M* halts in some other configuration

Physical Turing Machines



LEGO Turing Machine
https://www.youtube.com/watch?v=FTSAiF9AHN4

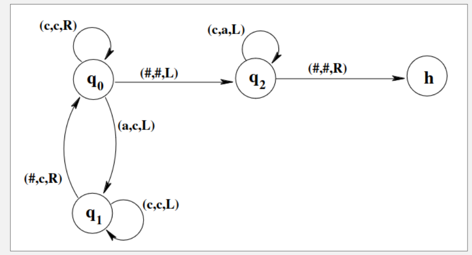


from the 2012 GO ASK A.L.I.C.E. exhibition at Harvard University's Collection of Historical Scientific Instruments

https://commons.wikimedia.org/wiki/File:Turing_Machine_Model_Davey_2012.jpg

Computing Functions

Definition 5.2. Suppose that Σ and Γ are alphabets that do not contain # and that f is a function from Σ^* to Γ^*. We say that f is ***Turing-computable*** if there is a Turing machine $M = (Q, \Lambda, q_0, \delta)$ such that $\Sigma \subseteq \Lambda$ and $\Gamma \subseteq \Lambda$ and for each string $w \in \Sigma^*$, when M is run with input w, it halts with output $f(w)$. In this case, we say that M ***computes*** the function f.



$\Sigma = \{ a \}$
computes $f(a^n) = a^{2n}$

1. Let $\Sigma = \{a\}$. Draw a transition diagram for a Turing machine that computes the function $f\colon \Sigma^* \to \Sigma^*$ where $f(a^n) = a^{3n}$, for $n \in \mathbb{N}$. Draw a transition diagram for a Turing machine that computes the function $f\colon \Sigma^* \to \Sigma^*$ where $f(a^n) = a^{3n+1}$, for $n \in \mathbb{N}$.

2. Let $\Sigma = \{a, b\}$. Draw a transition diagram for a Turing machine that computes the function $f\colon \Sigma^* \to \Sigma^*$ where $f(w) = w^R$.