

Show that the following argument is valid using a truth table.

$$\frac{p \rightarrow q}{p} \therefore q$$

Answer:

p	q	$p \rightarrow q$	$(p \rightarrow q) \wedge p$	$((p \rightarrow q) \wedge p) \rightarrow q$
true	true	true	true	true
true	false	false	false	true
false	true	true	false	true
false	false	true	false	true

Proving a conclusion is equivalent to saying that *the conjunction of the premises implies the conclusion* is a tautology; the last column of the truth table shows that this is the case.

Express the following English arguments in terms of propositional logic and determine whether the arguments are valid or invalid.

- (a) It if is Sunday, it rains or snows. Today, it is Sunday and it's not raining. Therefore, it must be snowing.

Answer: Let *sun* be "it is Sunday", *rain* be "it rains", and *snow* be "it snows". Then the argument can be expressed as

$$\frac{\begin{array}{l} sun \rightarrow (rain \vee snow) \\ sun \wedge \neg rain \end{array}}{\therefore snow}$$

1. $sun \rightarrow (rain \vee snow)$ premise
2. $sun \wedge \neg rain$ premise
3. sun from 2. (pg 40)
4. $rain \vee snow$ from 1., 3. (*modus ponens*)
5. $\neg rain \wedge sun$ from 2. (Commutative Law)
6. $\neg rain$ from 5. (pg 40)
7. $snow$ from 4., 6. (pg 40)