Question 5 dealt with expressing English statements in predicate logic.

- *Make sure you have quantifiers for variables.* Avoid unbound variables — predicates can only be applied to individual entities, so $P(x)$ doesn't make sense unless x is a specific entity. Instead, you need $\forall x P(x)$ or $\exists x P(x)$ to express that P is true for all or for some entities x in the domain of discourse.

- *Keep predicates simple; prefer less-specific predicates.* To express the idea "x is a black cat", define separate predicates $B(x)$ "x is black" and $C(x)$ "x is a cat" and write $B(x) \wedge C(x)$ instead of defining only a single predicate $BC(x)$ "x is a black cat". A predicate is atomic — you can't break up a single predicate into smaller pieces, and so the only way to express the notion that "is a black cat" means "is black" and "is a cat" is to have separate predicates. Otherwise "black cat" is a single, nondivisible thing.

- *Prefer a predicate to limiting the domain of discourse.* Adjectives should generally be predicates instead of being expressed by limiting the domain of discourse. For example, expressing "black cats are unlucky" as $\forall x(B(x) \rightarrow U(x))$ where $B(x)$ is "is black", $U(x)$ is "is unlucky", and the domain is discourse is cats is better than as $\forall x U(x)$ where the domain of discourse is black cats.

  In addition, prefer additional predicates when the commonly-understand domain of discourse for a predicate is larger than you need. For example, "is unlucky" can apply to a lot more than cats, so it would be better still to express "black cats are unlucky" as $\forall x((B(x) \wedge C(x)) \rightarrow U(x))$ where $C(x)$ is "is a cat" — for all things x, if x is black and x is a cat, then x is unlucky.

- *Include enough places in the predicate.* "Owns" involves both an owner and a thing being owned, and so should have placeholders for both of these things. For example, to express "everyone who owns a black cat is unlucky", the predicate should be $O(x, y)$ "x owns y": $\forall x \forall y((O(x, y) \wedge B(y) \wedge C(y)) \rightarrow U(x))$ captures "for all things x and y, if y is a black cat and x owns y, then x is unlucky" or "everyone who owns a black cat is unlucky".

  Similarly, "read" involves both a book and a reader — $R(x, y)$ means "x has read y".

- *Too much in the predicate.* The two-place predicate $O(x, y)$ "x owns y" is preferable to a one-place predicate such as $OBC(x)$ "x owns a black cat" because the one-place predicate doesn't allow quantifiers or other predicates to be applied to the cat.

  Similarly, $R(x, y)$ "x has read y" is better than $NR(x)$ "no one has read x". "No one" is something that should involve a quantifier — $\forall x \neg R(x, b)$ for "for all x, x has not read b" or "no one has read b". (b is a specific book.)

- *Separate variables for separate things.* Make sure there are separate variables for separate things. For example, let $P(x)$ be "x is a problem" and $S(x)$ be "x is a solution". $\forall x(P(x) \rightarrow S(x))$ doesn't make sense — for all things x, if x is a problem then x is a solution. (Something is either a problem or a solution, not both.) $\forall x(P(x) \rightarrow \exists y S(y))$ is getting closer — x is a problem, y is a solution — but is missing that y needs to be a solution for x. $\forall x(P(x) \rightarrow \exists y S(x, y))$ where $S(x, y)$ is "y is a solution for problem x" successfully captures "for every x, if x is a problem, there exists a y that is a solution for that problem" or "every problem has a solution".

- *Recognize specific entities.* "There is a solution for this problem" refers to a particular entity — "this problem". This is distinct from a statement like "there is a solution for every problem". In the latter case, an entity variable is used for the problem: $\forall x(P(x) \rightarrow \exists y S(x, y))$ or "every problem has a solution". For the first statement, $\exists y S(\text{this problem}, y)$ or, if you prefer a shorter statement, $\exists y S(p, y)$ where p is this problem. Both of these express the statement "there exists a solution for this problem" or "this problem has a solution".