To show that an argument is valid, you need to show that the conjunction of the premises implying the conclusion is a tautology. For #1, that means showing that $(p \to q) \wedge (\neg p \to q) \to q$ is a tautology.

#1 also asked for an explanation of why the argument makes sense — this isn't asking for only an English translation of the notation, but rather for an understanding of what that combination of premises means. (p and $\neg p$ are the only two possibilities, so if q is true in both cases, it's always be true.)

For formal proofs (#2, #3), common problems included not citing the rule justifying each step and combining multiple steps into one. In a few cases, invalid deductions were made, though this generally only occurred in cases where justifications weren't given — so be sure to include those justifications!

For #3, write down the argument in formal logic even if it is invalid, and then either give a proof or explain why it isn't valid — in some cases, only part of this was done.

When translating English to formal logic, you may have a choice between simple propositions or predicates. In #3a, all of the statements are about a single entity ("this card") rather than something that could apply to multiple entities of some type. Defining propositions like r for "this card is red" is reasonable.

In #3b, however, some of the statements are about "a math major" (so they apply to any person who is a math major) and some are about a specific person (Alice). So a direct translation of the problem statement would be to define predicates of the form $P(x)$ for statements about person x and write things like $P(a)$ for statements about a specific person a (Alice).

Also keep in mind that with variables like x, quantifiers are needed — $M(x)$ isn't by itself a proposition because x isn't a specific entity, so just writing something like $M(x) \to A(x) \wedge F(x)$ isn't quite right.

For #4–8, be sure to provide sufficient framing — write what you are trying to show (only if your proof goes immediately below the problem statement can you omit that statement) and what you are assuming. You also need to state what you are showing if you aren't doing a direct proof. Connect the dots — don't take too many deductive steps at once, provide justifications when bringing in external facts not well-known to the expected domain of readers of your proof, and explain why the last step in your chain of reasoning shows what you are trying to prove.

To prove an implication $p \to q$ using a direct proof, assume p and deduce q.

An indirect proof of $p \to q$ means to show the contrapositive $\neg q \to \neg p$: assume $\neg q$ and deduce $\neg p$.

Proving a proposition by contradiction means to assume the opposite and deduce a demonstrably false statement. For $p \to q$, that means to assume $\neg(p \to q) \equiv p \land \neg q$ and then deduce a false statement (which may be $\neg p$ or could be something else). Note that you must actually *use* $\neg q$ in your string of deductions — it is not a proof by contradiction to state that you are assuming $p \land \neg q$ but then proceed with deductions based on only p.

For #6, make sure you show both directions — iff ($\leftrightarrow$) means that you need both $p \to q$ and $q \to p$.

Be careful not to overlook details, whether part of assumptions or definitions. For example, the definition that says a rational number is one that can be written in the form $\frac{m}{n}$ also stipulates that m and n are integers and that $n \neq 0$. Knowing that about m and n may be important as you proceed forward, and if you use the definition in reverse (a number is rational because it can be written in the form $\frac{m}{n}$), make sure you can prove that m and n are integer and n is non-zero.