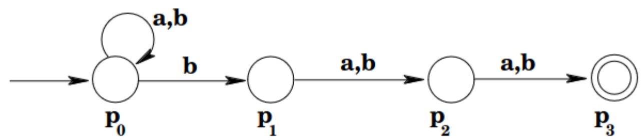
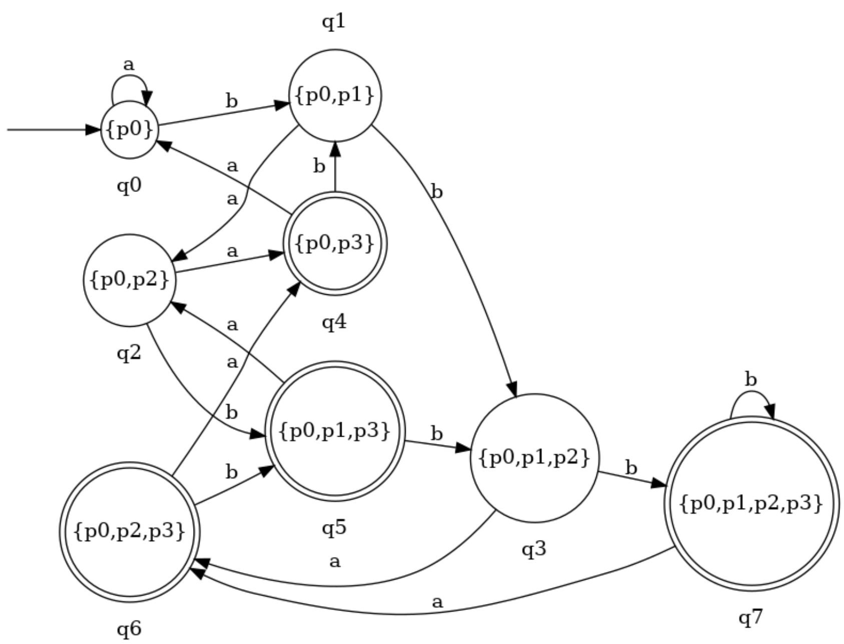
Give a DFA that accepts the language accepted by the following NFA.

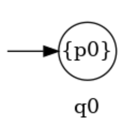


Answer:



Discussion: Let D be the DFA and N be the NFA.

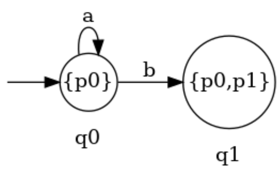
The start state of D corresponds to $\partial^*(p_0, \epsilon)$, that is, the set of states of N containing N's start state and everything reachable from that state via ϵ-transitions. (There are no ϵ-transitions here, so it is just $\{p_0\}$.)



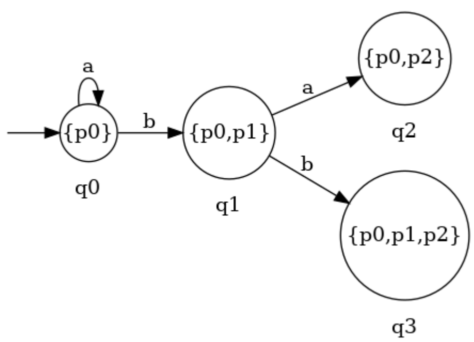
Now, repeatedly find a state q in D whose out-transitions haven't been added, and add them: for each input symbol a, look at all of N's states that can be reached from any one of the p_i corresponding to q by consuming a (including any subsequent

ϵ-transitions). Add state $q' = \bigcup \partial^*(p_i, a)$ if not already present and add transition $\delta(q, a) = q'$ to D.

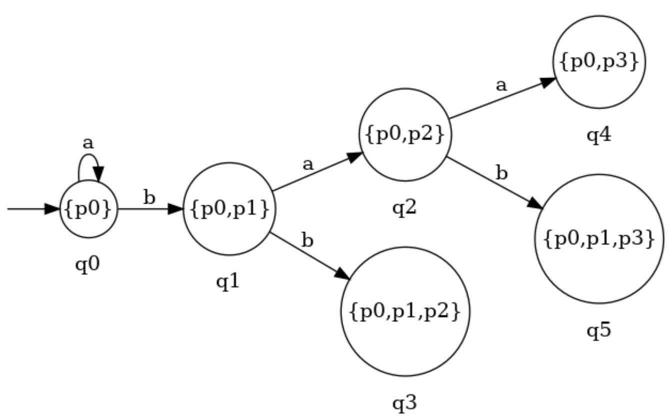
q_0 doesn't have out-transitions yet. From p_0, a goes to p_0, so add a transition $q_0 \to q_0$ to D. From p_0, b goes to p_0 or p_1, so add a state $\{p_0, p_1\}$ and a transition from q_0 to that state.



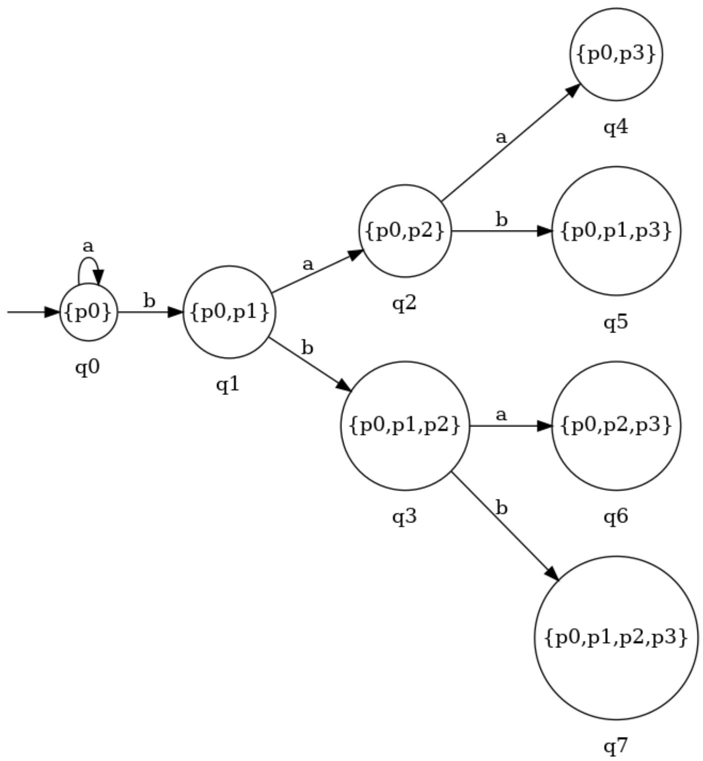
q_1 doesn't have out-transitions yet. From p_0, a goes to p_0 and from p_1, a goes to p_2. From p_0, b goes to p_0 or p_1 and from p_1, b goes to p_2.



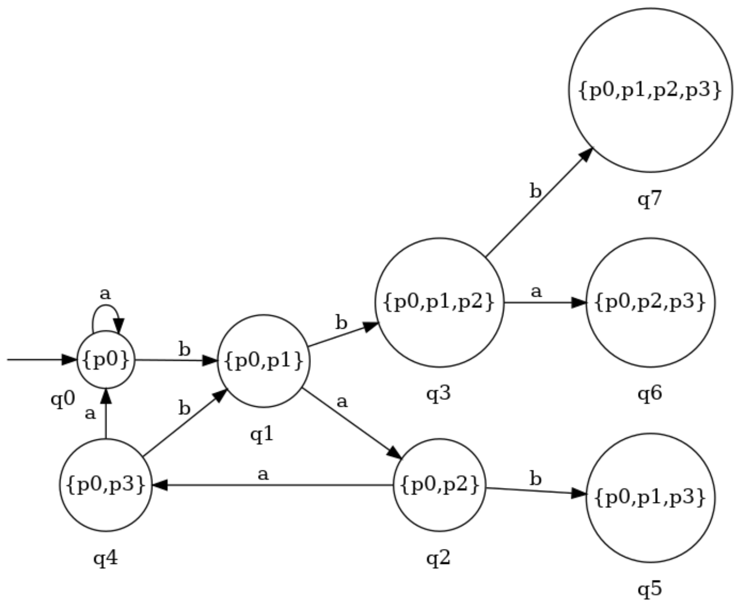
q_2 doesn't have out-transitions yet. From p_0, a goes to p_0 and from p_2, a goes to p_3. From p_0, b goes to p_0 or p_1 and from p_2, b goes to p_3.



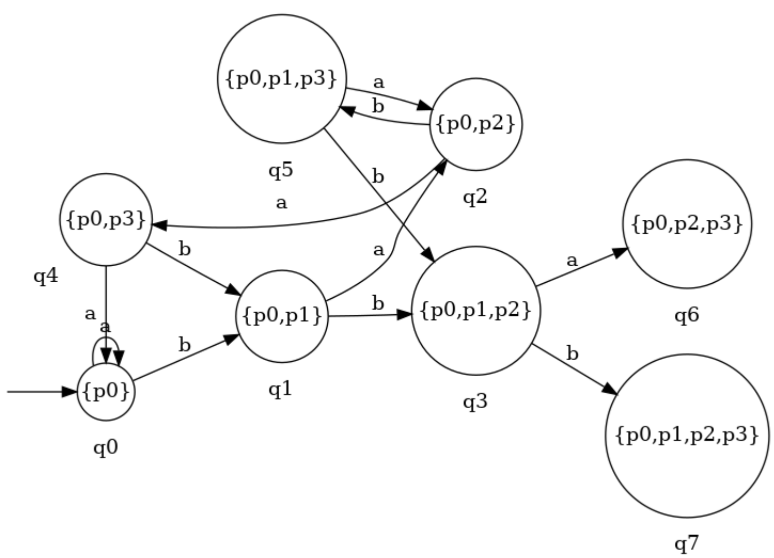
q_3 doesn't have out-transitions yet. From p_0, a goes to p_0, from p_1, a goes to p_2, and from p_2, a goes to p_3. From p_0, b goes to p_0 or p_1, from p_1, b goes to p_2, and from p_2, b goes to p_3.



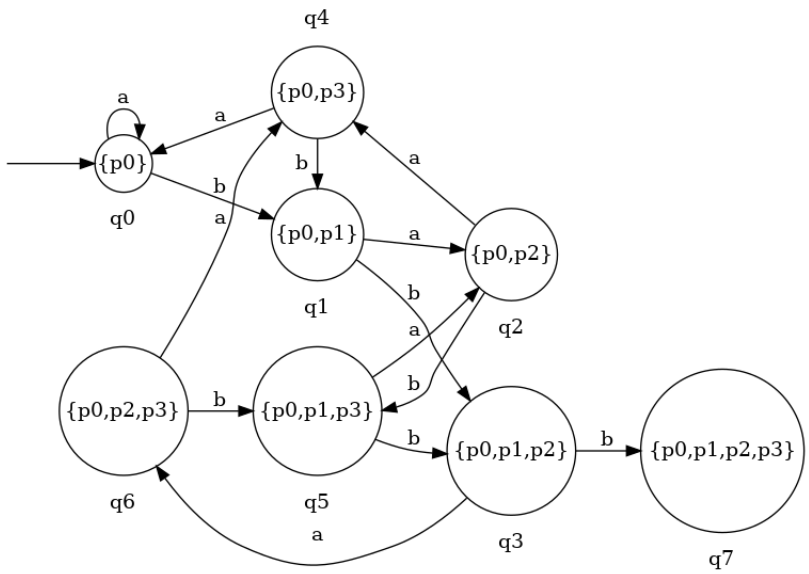
q_4 doesn't have out-transitions yet. From p_0, a goes to p_0, and from p_3, a goes nowhere. From p_0, b goes to p_0 or p_1, and from p_3, b goes nowhere.



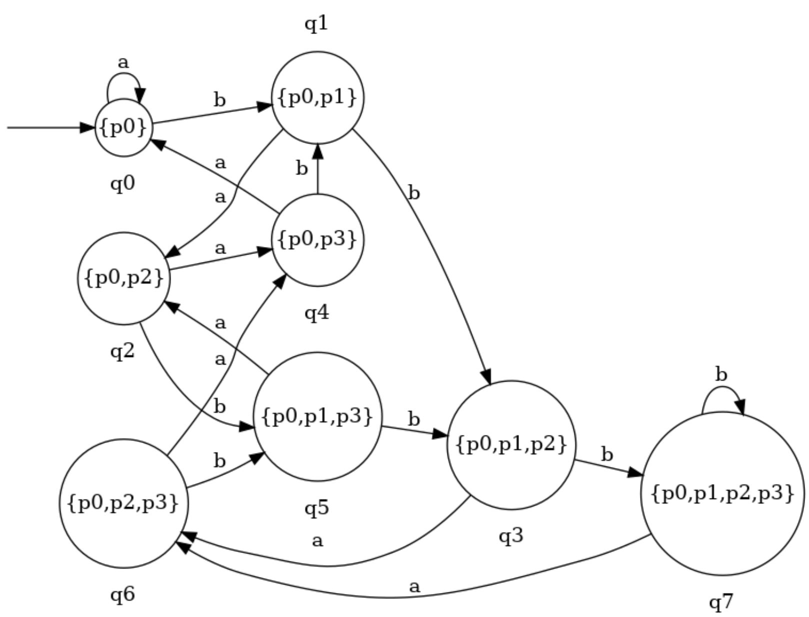
q_5 doesn't have out-transitions yet. From p_0, a goes to p_0, from p_1, a goes to p_2, and from p_3, a goes nowhere. From p_0, b goes to p_0 or p_1, from p_1, b goes to p_2, and from p_3, b goes nowhere.



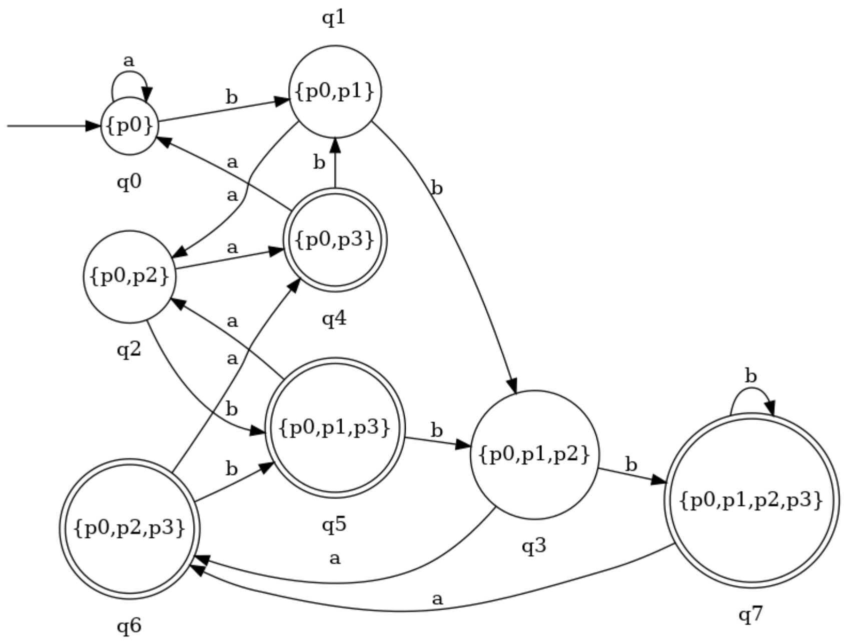
q_6 doesn't have out-transitions yet. From p_0, a goes to p_0, from p_2, a goes to p_3, and from p_3, a goes nowhere. From p_0, b goes to p_0 or p_1, from p_2, b goes to p_3, and from p_3, b goes nowhere.



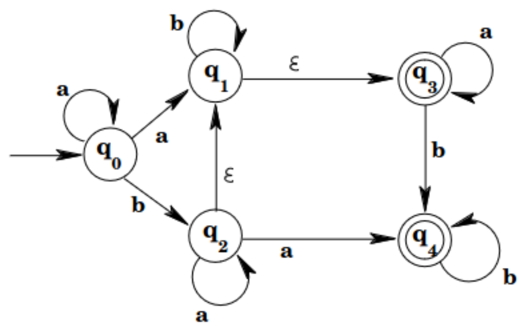
q_7 doesn't have out-transitions yet. From p_0, a goes to p_0, from p_1, a goes to p_2, from p_2, a goes to p_3, and from p_3, a goes nowhere. From p_0, b goes to p_0 or p_1, from p_1, b goes to p_2, from p_2, b goes to p_3, and from p_3, b goes nowhere.



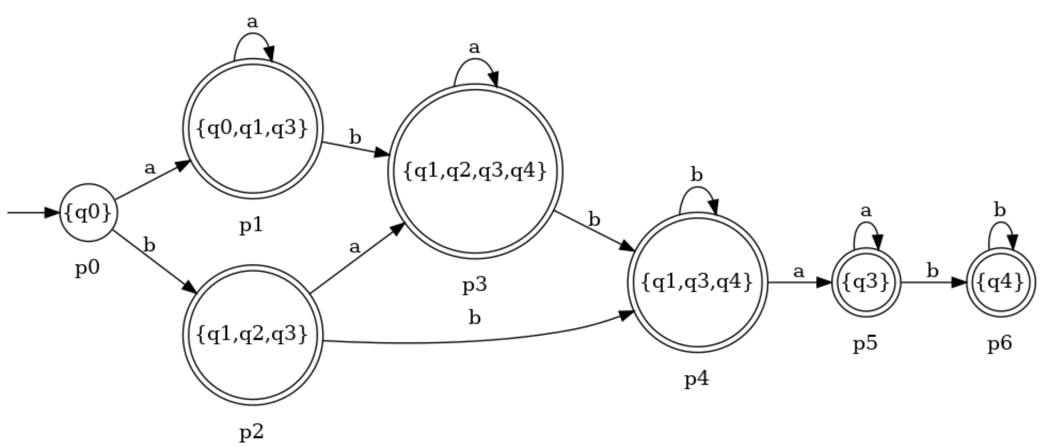
The final step is that any state of D containing a final state of N is a final state.



3. Give a DFA that accepts the language accepted by the following NFA. (Be sure to note that, for example, it is possible to reach both q_1 and q_3 from q_0 on consumption of an a, because of the ε-transition.)

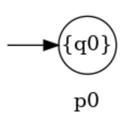


Answer:



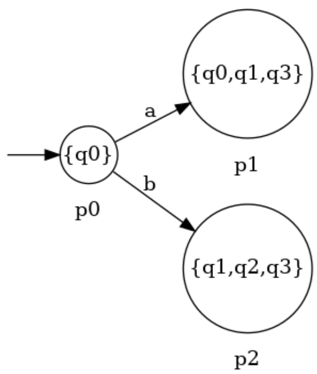
Discussion: Let D be the DFA and N be the NFA.

The start state of D corresponds to $\partial^*(q_0, \epsilon)$, that is, the set of states of N containing N's start state and everything reachable from that state via ϵ-transitions. (There are no ϵ-transitions from q_o, so it is just $\{q_0\}$.)

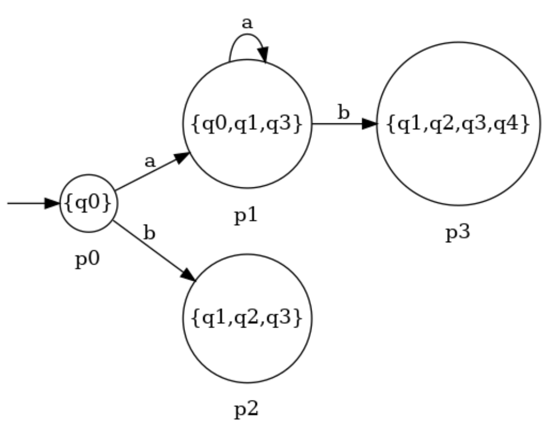


Now, repeatedly find a state p in D whose out-transitions haven't been added, and add them: for each input symbol a, look at all of N's states that can be reached from any one of the q_i corresponding to p by consuming a (including any subsequent ϵ-transitions). Add state $p' = \bigcup \partial^*(q_i, a)$ if not already present and add transition $\delta(p, a) = p'$ to D.

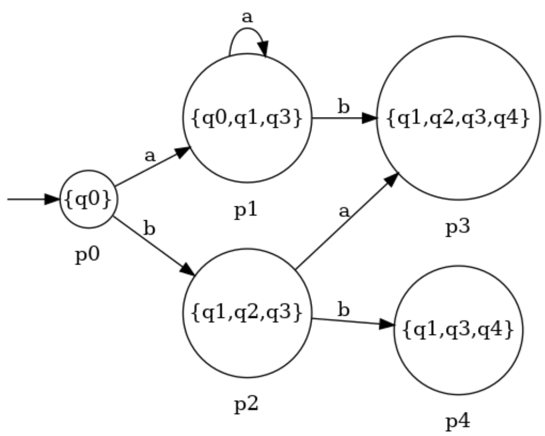
p_0 doesn't have out-transitions yet. From q_0, a goes to q_0 or q_1, then ϵ-transitions get to q_3. From q_0, b goes to q_2, then ϵ-transitions get to q_1 and q_3.



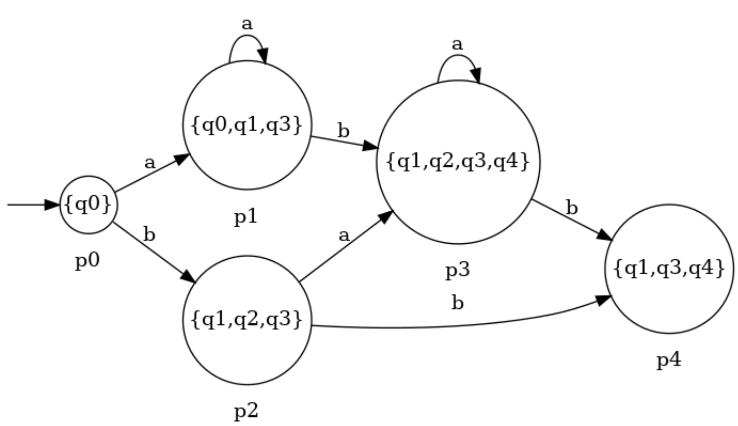
p_1 doesn't have out-transitions yet. From q_0, a goes to q_0 or q_1, then ϵ-transitions get to q_3; from q_1, a goes nowhere; and from q_3, a goes to q_3. From q_0, b goes to q_2, then ϵ-transitions get to q_1 and q_3; from q_1, b goes to q_1, then ϵ-transitions get to q_3; and from q_3, b goes to q_4.



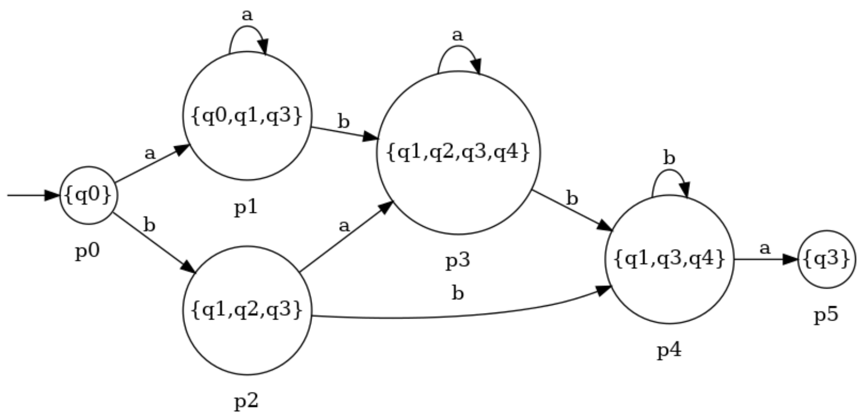
p_2 doesn't have out-transitions yet. From q_1, a goes nowhere; from q_2, a goes to q_2 or q_4, then ϵ-transitions get to q_1 and q_3; and from q_3, a goes to q_3. From q_1, b goes to q_1, then ϵ-transitions get to q_3; from q_2, b goes nowhere; and from q_3, b goes to q_4.



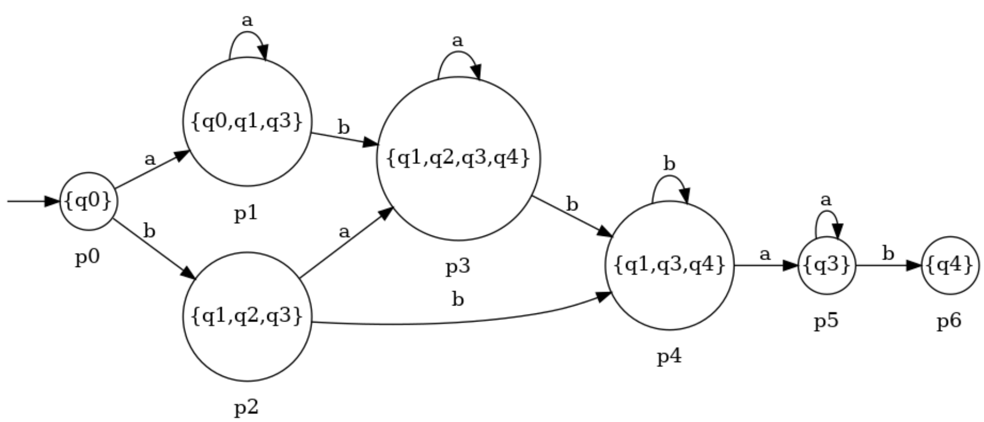
p_3 doesn't have out-transitions yet. From q_1, a goes nowhere; from q_2, a goes to q_2 or q_4, then ϵ-transitions get to q_1 and q_3; from q_3, a goes to q_3; and from q_4, a goes nowhere. From q_1, b goes to q_1, then ϵ-transitions get to q_3; from q_2, b goes nowhere; from q_3, b goes to q_4; and from q_4, b goes to q_4.



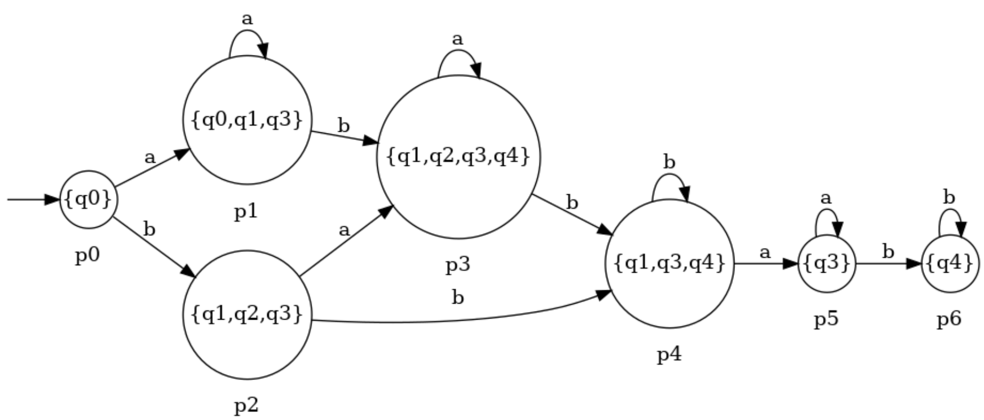
p_4 doesn't have out-transitions yet. From q_1, a goes nowhere; from q_3, a goes to q_3; and from q_4, a goes nowhere. From q_1, b goes to q_1, then ϵ-transitions get to q_3; from q_3, b goes to q_4; and from q_4, b goes to q_4.



p_5 doesn't have out-transitions yet. From q_3, a goes to q_3. From q_3, b goes to q_4.



p_6 doesn't have out-transitions yet. From q_4, a goes nowhere. From q_4, b goes to q_4.



The final step is that any state of D containing a final state of N is a final state.