Show that the following grammar is ambiguous by finding a string that has two left derivations according to the grammar.

$$
\begin{aligned}
S &\longrightarrow SS \\
S &\longrightarrow aSb \\
S &\longrightarrow bSa \\
S &\longrightarrow \epsilon
\end{aligned}
$$

Answer: $aabb$ is a string with two left derivations:

$$
\begin{aligned}
S &\Longrightarrow aSb \\
S &\Longrightarrow aaSbb \\
S &\Longrightarrow aabb
\end{aligned}
\qquad\qquad
\begin{aligned}
S &\Longrightarrow SS \\
&\Longrightarrow aSbS \\
&\Longrightarrow aaSbbS \\
&\Longrightarrow aabbS \\
&\Longrightarrow aabb
\end{aligned}
$$

Discussion: The goal is to find two left derivations that lead to the same string, so a strategy is to start off with applying different rules — thus the derivations will be different — and then try to get both derivations to the same string.

Start each derivation with a different rule:

$$
S \Longrightarrow aSb \qquad\qquad S \Longrightarrow bSa
$$

But we can see that in the first one, whatever string is derived will start with a and end with b, while the opposite is true in the second derivation. These derivations will never result in the same string.

Try something else —

$$
S \Longrightarrow aSb \qquad\qquad S \Longrightarrow SS
$$

Since the first derivation will result in a string starting with a and ending with b, we need to aim for that in the second derivation as well.

$$
S \Longrightarrow aSb
\qquad\qquad
\begin{aligned}
S &\Longrightarrow SS \\
&\Longrightarrow aSbS
\end{aligned}
$$

Now apply the same steps to each derivation.

$$
\begin{aligned}
S &\Longrightarrow aSb \\
S &\Longrightarrow aaSbb \\
S &\Longrightarrow aabb
\end{aligned}
\qquad\qquad
\begin{aligned}
S &\Longrightarrow SS \\
&\Longrightarrow aSbS \\
&\Longrightarrow aaSbbS \\
&\Longrightarrow aabbS
\end{aligned}
$$

Finally, eliminate the final S on the right side.

$$
\begin{aligned}
S &\Longrightarrow aSb \\
S &\Longrightarrow aaSbb \\
S &\Longrightarrow aabb
\end{aligned}
\qquad\qquad
\begin{aligned}
S &\Longrightarrow SS \\
&\Longrightarrow aSbS \\
&\Longrightarrow aaSbbS \\
&\Longrightarrow aabbS \\
&\Longrightarrow aabb
\end{aligned}
$$

Find a left derivation for $(x + y) * z$ in the following grammar.

$$\begin{aligned} E &\longrightarrow E \ + \ T \\ E &\longrightarrow T \\ T &\longrightarrow T \ * \ F \\ T &\longrightarrow F \\ F &\longrightarrow (E) \\ F &\longrightarrow x \\ F &\longrightarrow y \\ F &\longrightarrow z \end{aligned}$$

Answer:

$$\begin{aligned} E &\Longrightarrow T \\ &\Longrightarrow T * F \\ &\Longrightarrow F * F \\ &\Longrightarrow (E) * F \\ &\Longrightarrow (E + T) * F \\ &\Longrightarrow (T + T) * F \\ &\Longrightarrow (F + T) * F \\ &\Longrightarrow (x + T) * F \\ &\Longrightarrow (x + F) * F \\ &\Longrightarrow (x + y) * F \\ &\Longrightarrow (x + y) * z \end{aligned}$$

Discussion: What is interesting here is not the derivation itself, but the process — the first decision, for example, is between $E \longrightarrow E + T$ and $\longrightarrow T$. Which to choose? Just looking at the first symbol $(x + y) * z$ isn't enough.