## Graph ADT Implementations

<u>adjacency matrix</u>

graph stores
- a list of vertices
- a list of edges
- **2D array, indexed by vertex key**

vertex stores
- the associated object
- degree of the vertex
- **distinct integer key in the range 0..n-1**

edge stores
- the associated object
- endpoint vertices

**array stores**
- **A[i][j] holds the edge from vertex with index i to vertex with index j (`null` if no edge)**

<u>adjacency list</u>

graph stores
- a list of vertices
- a list of edges

vertex stores
- the associated object
- degree of the vertex
- **list of incident edges**

edge stores
- the associated object
- endpoint vertices

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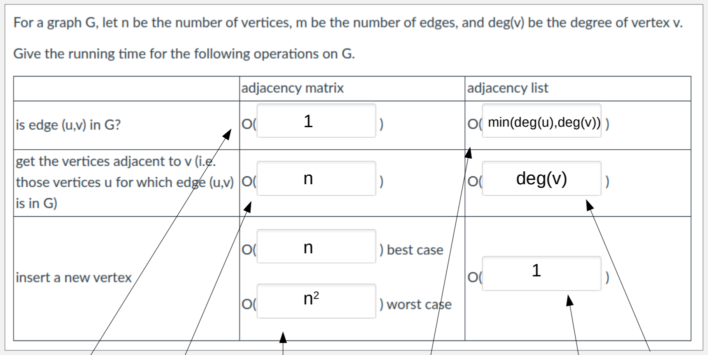

For a graph G, let n be the number of vertices, m be the number of edges, and deg(v) be the degree of vertex v.

Give the running time for the following operations on G.

| | adjacency matrix | adjacency list |
|---|---|---|
| is edge (u,v) in G? | O( 1 ) | O( min(deg(u),deg(v)) ) |
| get the vertices adjacent to v (i.e. those vertices u for which edge (u,v) is in G) | O( n ) | O( deg(v) ) |
| insert a new vertex | O( n ) best case<br>O( $n^2$ ) worst case | O( 1 ) |

access `array[u.key][v.key]`

must scan through entire row (col) of array

best case doesn't need growing, but must initialize row and col for new vertex

search through one vertex's adjacency list – pick the one with smaller degree

search v's adjacency list

add to (unordered) list of vertices

## Adjacency Matrix Implementation

graph stores
- a list of vertices
- a list of edges

} **doubly-linked list allows for O(1) removal given reference to list node**

- 2D array, indexed by vertex key

vertex stores
- the associated object
- degree of the vertex
- **reference to the vertex's location in the list of vertices**
- distinct integer key in the range 0..n-1

edge stores
- the associated object
- endpoint vertices
- **reference to the edge's location in the list of edges**

array stores
- A[i][j] holds the edge from vertex with index i to vertex with index j (null if no edge)

## Adjacency List Implementation

graph stores
- a list of vertices
- a list of edges

} **doubly-linked list allows for O(1) removal given reference to list node**

vertex stores
- the associated object
- degree of the vertex
- **reference to the vertex's location in the list of vertices**
- list of incident edges } **doubly-linked list allows for O(1) removal given reference to list node**

edge stores
- the associated object
- endpoint vertices
- **reference to the edge's location in the list of edges**
- **references to the edge's location in the incidence lists for its endpoint vertices**

| | adjacency list | adjacency matrix |
|---|---|---|
| numVertices(), numEdges() | O(1) | O(1) |
| vertices(), edges() | O(1) per element | O(1) per element |
| aVertex() | O(1) | O(1) |
| degree(v) | O(1) | O(1) |
| adjacentVertices(v) | O(1) per element | O(n) – to scan row/column of array |
| incidentEdges(v) | O(1) per element | O(n) – to scan row/column of array |
| endVertices(e) | O(1) | O(1) |
| opposite(v,e) | O(1) | O(1) |
| areAdjacent(v,w) | O(min(deg(v,w))) – search list for vertex with smaller degree | O(1) |
| insertEdge(v,w,o) | O(1) | O(1) |
| insertVertex(o) | O(1) | O(n) – to initialize row/col of array<br>O($n^2$) – if array needs to grow |
| removeVertex(v) | O(deg(v)) – to remove each incident edge | O(1) – with clever bookkeeping (and wasted space)<br>O($n^2$) – shifting in array |
| removeEdge(e) | O(1) | O(1) |
| space | O(n+m) | O($n^2$) |

## Comparison

Adjacency matrix –

- **very time-efficient for `isAdjacent` – O(1)**
- very space-inefficient for sparse graphs
- time-inefficient for traversing edges incident on a vertex – O(n)
- time-inefficient for insert/remove vertex

Adjacency list –

- **space-efficient except for the most dense graphs**
- **time-efficient for traversing edges incident on a vertex – O(deg)**
- `isAdjacent` is O(deg) rather than O(1)