Greedy Algorithms

- iterative series-of-choices
  - main steps: repeat make the next choice until done
- always make a *local* decision
  - each choice is made without consideration of future possibilities
  - pick a single alternative for each choice, no regrets – can't reconsider later
- often, but not exclusively, applied to optimization problems
  - goal is to find the best solution among (generally) many legal solutions
  - for non-optimization problems, goal is to find a legal solution among (generally) many invalid (non-)solutions
- don't work for everything
  - some problems aren't solvable with only locally good decisions
    - e.g. knapsack – empty space has no value, so it may be better to take several lesser-valued items that fill more of the space and thus achieve a higher total value
- a correctness proof is essential!
  - finding counterexamples is an important technique for identifying incorrect greedy choices

Boston to Seattle

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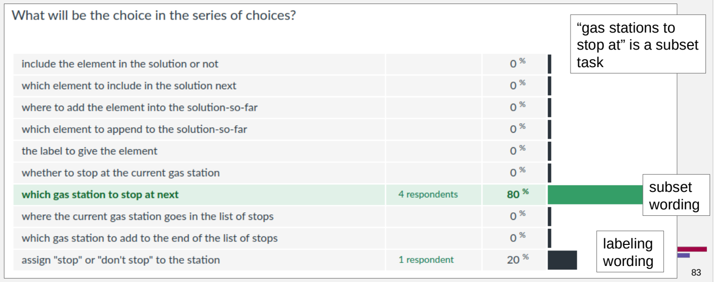
- series-of-choices patterns and flavors

task	iterative pattern	choice
subset	process input	include element in the solution or not
	produce output	which element to include in the solution next
ordering	process input	where to add the element into the solution-so-far
	produce output	which element to append to the solution-so-far
labeling	process input	the label to give the element



What will be the choice in the series of choices?

include the element in the solution or not		0 %	
which element to include in the solution next		0 %	
where to add the element into the solution-so-far		0 %	
which element to append to the solution-so-far		0 %	
the label to give the element		0 %	
whether to stop at the current gas station		0 %	
which gas station to stop at next	4 respondents	**80 %**	subset wording
where the current gas station goes in the list of stops		0 %	
which gas station to add to the end of the list of stops		0 %	
assign "stop" or "don't stop" to the station	1 respondent	20 %	labeling wording

"gas stations to stop at" is a subset task

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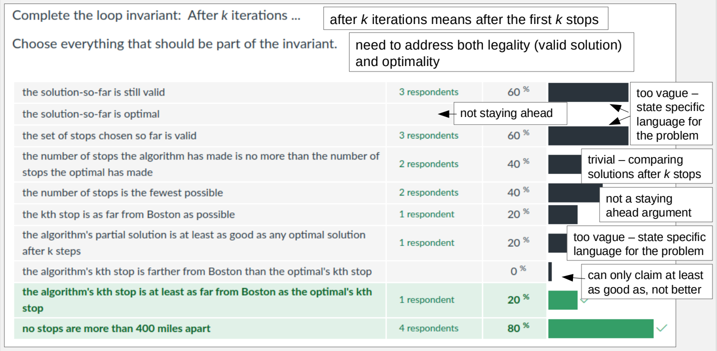
- main steps: repeat pick which gas station to stop at next until we get within 400 miles of Seattle



Complete the loop invariant: After *k* iterations ...

after *k* iterations means after the first *k* stops

Choose everything that should be part of the invariant.

need to address both legality (valid solution) and optimality

the solution-so-far is still valid	3 respondents	60 %	too vague – state specific language for the problem
the solution-so-far is optimal			not staying ahead
the set of stops chosen so far is valid	3 respondents	60 %	
the number of stops the algorithm has made is no more than the number of stops the optimal has made	2 respondents	40 %	trivial – comparing solutions after *k* stops
the number of stops is the fewest possible	2 respondents	40 %	not a staying ahead argument
the kth stop is as far from Boston as possible	1 respondent	20 %	
the algorithm's partial solution is at least as good as any optimal solution after k steps	1 respondent	20 %	too vague – state specific language for the problem
the algorithm's kth stop is farther from Boston than the optimal's kth stop		0 %	can only claim at least as good as, not better
the algorithm's kth stop is at least as far from Boston as the optimal's kth stop	1 respondent	**20 %** ✓	
no stops are more than 400 miles apart	4 respondents	**80 %** ✓	