

This homework covers reductions to show hardness. It is due in class Monday, May 4.

See the Policies page on the course website for information about revise-and-resubmit, late work, and academic integrity as it applies to homework.

Write your solutions carefully — your work should be neat, readable, and organized. Keep in mind that what you hand in should be a presentation of your work, not your train-of-thought scratch work with a solution mixed in somewhere.

1. Do the homework #13 drill problems on Canvas. (Look for `hw13 drill` in the Quizzes section.)
2. The vertex cover problem is known to be NP-complete. Prove that the card collector problem is also NP-complete using a reduction. Keep in mind that you need to show that the card collector problem is in NP as well as giving the reduction itself. Also explain why the reduction is correct — you'll get the right answer to the problem — and polynomial-time.

The vertex cover and card collector problems are described below.

Vertex cover Given a graph G and a positive integer k , is there a vertex cover of G with size at most k ? A *vertex cover* of a graph G is a set of vertices such that every edge of G has at least one endpoint in the set.

Card collector Given n packets of cards (each of which contains a subset of the available cards) and a positive integer k , is it possible to collect the full set of cards by buying no more than k packets? For example, if there are four cards available (A, B, C, D) and the packets contain $\{A,B\}$, $\{B,C\}$, $\{D\}$, $\{B,D\}$, then the answer is *no* for $k=2$ (packet 1 is needed for card A, packet 2 is needed for card C, and either packet 3 or 4 is needed for card D) and *yes* for $k=3$ (e.g. packets $\{A,B\}$, $\{B,C\}$, $\{D\}$ result in the full set).