

Splay Trees



- invented by Daniel Sleator and Robert Tarjan in 1985

A *splay tree* is a BST + a restructuring operation:

- after each find/insert/remove, that node (or its parent) is brought to the root through *splaying*

Observation.

- frequently-accessed nodes are near the root

Does this ensure $O(\log n)$ height?

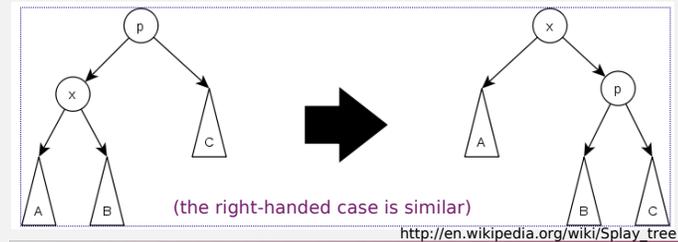
- on average, yes
- worst case is $O(n)$ – but the worst case is unlikely

Splaying

- x is the node being splayed
- p is the parent of x
- g is the parent of p (i.e. the grandparent of x)

Case 1: zig – applies when p is the root

- x is rotated to the root

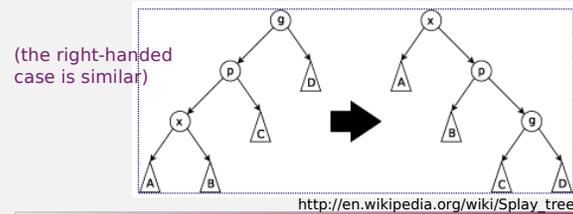


Splaying

- x is the node being splayed
- p is the parent of x
- g is the parent of p (i.e. the grandparent of x)

Case 2: zig-zig – applies when p is not the root, and x and p are both either right children or left children

- p is rotated into g 's position, then x is rotated into p 's position

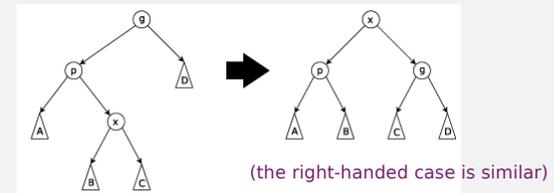


Splaying

- x is the node being splayed
- p is the parent of x
- g is the parent of p (i.e. the grandparent of x)

Case 3: zig-zag – applies when p is not the root, and one of x and p is a right child and the other is a left child

- x is rotated into p 's position, then x is rotated into g 's position



Performance

- all operations are $O(\text{height})$ to perform the operation + $O(\text{height})$ splay steps
 - each zig-zig or zig-zag raises x two levels, each zig (done at most one per splay) raises x one level
 - $O(\log n)$ amortized
- worst-case performance
 - splay trees perform as well as optimum static balanced BSTs on sequences of at least n accesses (up to a constant factor)
 - “static” = no restructuring of tree after construction
 - “optimal” = tree providing smallest possible time for a series of accesses
 - it is conjectured that splay trees perform as well as optimum dynamic balanced BSTs on sequences of at least n accesses (up to a constant factor)
 - “dynamic” = tree can be restructured after construction (e.g. AVL trees, red-black trees)

Splay Trees Takeaways

- another form of restructuring operation
- randomized or heuristic approaches can result in good performance in practice because worst case scenarios are rare
- the notion of amortized analysis
 - an average based on the performance over a series of operations