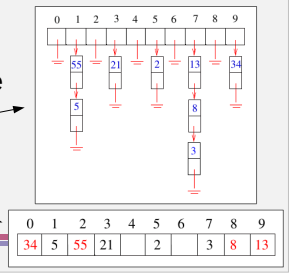
Hashtables Recap

- a good hash function h(k) –
  - is efficient to compute
    - ideally time is not dependent on |*k*| or *n*, but dependent on |*k*| is often effectively Θ(1)
  - spreads out the keys as much as possible
    - equal probability of h(k) = each value 0..N-1
- even with a good hash function, *collisions* (different keys hashing to the same array index) are inevitable
- two solutions
  - separate chaining
  - open addressing



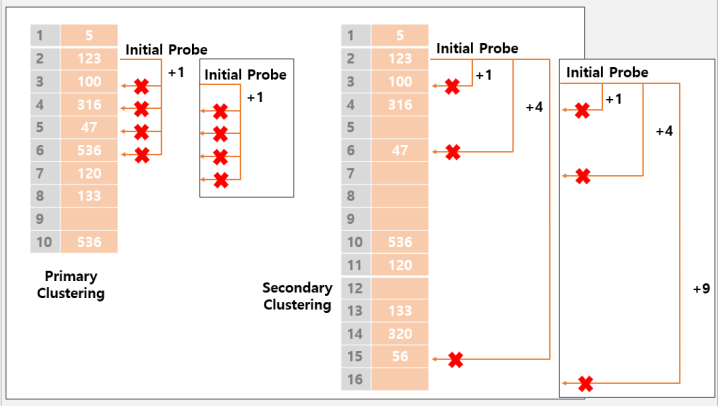
Open Addressing

- requires n ≤ N

If h(k) is full, follow a *probe sequence* to locate element / find first empty slot for insertion.

- linear probing – h(k) + c·i [c is often 1]
  - c should be relatively prime to N (not a problem if N is prime)
  - *sequential probing* when c=1
- quadratic probing – h(k) + i^2
- double hashing – h(k) + i h'(k)

Primary and Secondary Clustering



Open Addressing

- linear probing – h(k) + c·i [c is often 1]
  - exhibits better memory locality than other options
  - suffers from clustering
    - keys that hash to the same index or adjacent indexes interfere with each other
    - performance degrades quickly as n approaches N
  - sensitive to key distribution
    - uneven key distribution exacerbates the clustering problem
- quadratic probing – h(k) + i^2
  - suffers from secondary clustering
    - keys that hash to adjacent slots have adjacent probe sequences
  - may not find an empty slot even if one exists
- double hashing – h(k) + i h'(k)
  - expected length of unsuccessful probe sequence is 1/(1-α) → O(1) if table is not too full
    - α = n/N (load factor)

Open Addressing

Deletion requires special handling.

- can re-insert all elements following the deleted element
  - if the load factor is low enough, this should only be a small number of elements
  - only possible with sequential probing
- can mark empty slot as “deleted” – find continues on, insert can fill
  - drawback: probe sequence lengths are based on the largest the collection has been, not the current size
  - solution: can periodically re-hash everything to clean up

Hashtables

If done properly, hashtables provide O(1) expected time for find, insert, remove – once h(k) has been computed.

- “done properly” means load factor isn't too high and is kept bounded, and there is good distribution of hash values

Computing h(k) can take time.

- e.g. for strings, computing h(k) = O(|k|) … which reduces to O(1) if |k| is bounded, but must be considered as O(|k|) otherwise

Worst-case behavior is O(n) for find and remove, unless separate chaining + a fancier bucket implementation is used (which has memory overhead).

- worst case occurs when key distribution is poor and load factor is high

Hashtables

What about other operations?

- initialization
  - O(N) – size of the array used for the hashtable
- traversal
  - in most cases O(n+N) for separate chaining – must examine each index of table as well as all elements
    - can be worse e.g. worst case dynamic perfect hashing
  - O(N) for open addressing
- find next larger/smaller key, find min/max key
  - full traversal is required because h(k) does not preserve original ordering of keys

Map/Dictionary Implementation Recap

Dictionary operation	Unsorted array	Sorted array	Singly linked unsorted	Singly linked sorted	balanced BST	hashtable
Search(A, k)	$O(n)$	$O(\log n)$	$O(n)$	$O(n)$	O(log n)	O(1) expected
Insert(A, x)	$O(1)$	$O(n)$	$O(1)$	$O(n)$	O(log n)	O(1) expected
Delete(A, x) or Delete(A,k) (given location of *x*)	$O(1)^*$	$O(n)$	O(1) *	O(1) *	O(log n)	n/a
Remove(A,x) or Remove(A,k) (not given location of *x*)	O(n) requires search + delete	O(n)	O(n)	O(n)	O(log n)	O(1) expected

OrderedDictionary

Ordered Dictionary operation	Unsorted array	Sorted array	Singly linked unsorted	Singly linked sorted	balanced BST	hashtable
Search(A, k)						
Insert(A, x)						
Delete(A, x)						
Successor(A, x)						
Predecessor(A, x)						
Minimum(A)						
Maximum(A)						