

BFS-Based Algorithms

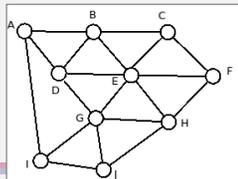
- reachability
 - can you get from u to v ?
 - what vertices can you get to from u ?
- unweighted shortest path
 - distance from u to all other vertices, in terms of the number of edges in the path

bfs(u)

- is u processed?
- processed vertices are reachable

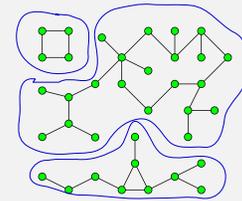
bfs(u)

- set $\text{dist}[v] = \text{dist}[u] + 1$ when v is discovered



BFS-Based Algorithms

a graph with three connected components (circled)



- connected components
 - a *connected component* is a subgraph where every pair of vertices are connected by a path and there are no connections to other vertices not in the subgraph
- ```

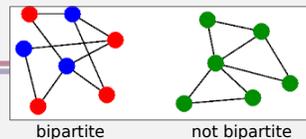
c = 0
for each vertex v of graph G
 if v has not been discovered
 run bfs(v), setting comp[u] = c when each vertex u is
 processed
 c++

```

intuition – BFS finds all vertices reachable from  $v$  along a path

## BFS-Based Algorithms

- bipartite graph detection / two-coloring



- a *bipartite* graph is one whose vertices can be divided into two sets such that every edge connects a vertex in one set with a vertex in the other
  - *coloring* refers to assigning labels (colors) to vertices so that no two adjacent vertices have the same label (color)
    - a *two-coloring* uses two colors
- ```

color[s] = 0
run bfs(s), setting color[v] = the opposite color of
  color[u] for each discovery edge (u,v) and checking that
  color[v] is the opposite color of color[u] for each non-
  discovery edge (u,v)
  – if there is an edge (u,v) for which color[u] = color[v], the graph is
    not bipartite / two-colorable
    
```

intuition – following a path along discovery edges must alternate colors, since those edges are graph edges

- can't change the color of any vertex without changing them all
- non-discovery edges are also graph edges, and ends must be opposite colors

Takeaways

- BFS algorithm
- BFS-based algorithms
 - graph traversal
 - reachability
 - unweighted shortest path
 - connected components
 - 2-coloring / detecting bipartite graphs