*This homework covers the reading for the third week of classes: Chapter 1, Sections 6, 7, and 8. It is due in class on Friday, February 9. You can work on these exercises with other people in the class, but you should write up your solutions in your own words to turn in. Remember that unsupported answers will receive little or no credit.*

*For the proofs that you are asked to give on this homework, you should give informal, but careful and complete, proofs of the kind that are typically given by mathematicians. In your proofs, you can use the following facts without proving them:*

- *The product of any two rational numbers is rational.*
- *The sum of any two rational numbers is rational.*
- *The numbers $\pi$ and $\sqrt{2}$ are irrational.*

*You can also use basic facts from algebra.*

**1.** Prove that for any integer $n$, the number $n^2 + n$ is even. (Consider a proof by cases, looking at the case where $n$ is even and the case where $n$ is odd.)

**2.** Suppose that $x$, $y$, and $z$ are integers such that $x + y + z$ is greater than 30. Prove that at least one of $x$ or $y$ or $z$ is greater than 10. (Consider a proof by contradiction.)

**3.** Prove: For any real number $x$, if $x^2$ is an irrational number, then $x$ is also irrational.

**4.** Disprove: For any real number $x$, if $x$ is an irrational number, then $x^2$ is also irrational.

**5.** Prove: For any real number $x$, either $x$ or $\pi - x$ is irrational.

**6.** Prove: Let $a$, $b$, and $c$ be integers, and assume that $b$ is divisible by $a$ and that $c$ is divisible by $b$. Then $c$ is divisible by $a$.

**7.** Write out each of the following sums in full, without using summation notation. You are **not** being asked to compute the value of the sums!

**a)** $\displaystyle\sum_{n=1}^{5} 3^n$ **b)** $\displaystyle\sum_{j=1}^{7} (2j-1)$ **c)** $\displaystyle\sum_{k=3}^{6} \frac{k}{k^2+1}$

**8.** Use a proof by induction to show that for any integer $n \geq 1$, $\displaystyle\sum_{i=1}^{n} (2i-1) = n^2$

**9.** Use a proof by induction to show that for any integer $n \geq 1$, $\displaystyle\sum_{i=1}^{n} \frac{1}{2^i} = 1 - \frac{1}{2^n}$

**10.** Use a proof by induction to show that for any integer $n \geq 1$, $n^3 + 2n$ is divisible by 3.