CPSC 229, Spring 2021 **Homework #9**

This homework covers the reading from Chapter 3, Section 7 and Chapter 4, Sections 1, 2, and 3. It is due by midnight on Monday, April 12, and will be accepted late with a 10% penalty until noon on Sunday, April 18. The second test will be given in class on Wednesday, April 14. It will cover Chapter 3 and Sections 4.1 through 4.3. It would be a good idea to complete this assignment before the test!

1. (*5 points*) Use the Pumping Lemma for Regular Languages to prove that the following languages are **not** regular. (Remember that any pumping lemma proof follows a similar pattern to the one given in class, starting with, "Suppose that L **is** regular. Then by the pumping lemma, there is an integer K such that if w is any string $w \in L$ with $|w| \geq K$, then w can be written $w = xyz$ where $|xy| \leq K$, $|y| \geq 1$, and $xy^nz \in L$ for all natural numbers n. But let $w = \dots$, which is in L and $|w| \geq K$.")

a) $L = \{ww \mid w \in \{a,b\}^*\}$. L consists of strings of a's and b's where the first half of the string is identical to the second half, such as $abbababbab$.

b) $L = \{w \in \{a,b\}^* \mid n_a(w) < n_b(w)\}$, where $n_\sigma(w)$ means the number of σ's in w.

2. (*5 points*) Consider the context free grammar shown at the right.

$$\begin{aligned} S &\longrightarrow TR \\ T &\longrightarrow aTb \\ T &\longrightarrow \varepsilon \\ R &\longrightarrow cRd \\ R &\longrightarrow c \end{aligned}$$

a) Write a derivation for the string $aabbc$ using this grammar.

b) Write a derivation for the string $abcccdd$ using this grammar.

c) Find the language generated by this grammar. Briefly justify your answer.

3. (*10 points*) For each of the following languages, create a Context-Free Grammar that generates that language. **Explain in words why your grammar works**. (As a hint for part (b), think about what you need to add to the grammar that we did in class for $\{w \in \{a,b\}^* \mid n_a(w) = n_b(w)\}$. As a hint for part (d), note that letters can match in pairs a/c, a/d, b/c, and b/d.)

a) $\{a^nba^n \mid n \in \mathbb{N}\}$

b) $\{w \in \{a,b]^* \mid n_a(w) < n_b(w)\}$

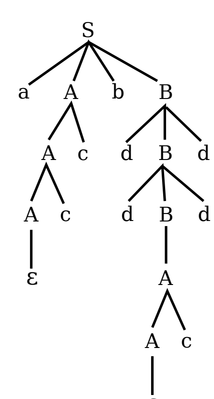
c) $\{a^nb^mc^kd^l \mid m = k \text{ and } n = l\}$

d) $\{a^nb^mc^kd^l \mid n + m = k + l\}$

4. (*3 points*) Given the following (very incomplete) BNF grammar for "names" in Java, write down **six** "names" generated by this grammar. Your examples should demonstrate all the possibilities represented in the rules.

$$\begin{aligned} \langle \textit{name} \rangle &::= \langle \textit{object\_ref} \rangle \ [\ \text{“.”}\ \langle \textit{identifier} \rangle\ ] \\ \langle \textit{object\_ref} \rangle &::= \langle \textit{identifier} \rangle \mid \langle \textit{method\_call} \rangle \\ \langle \textit{identifier} \rangle &::= \text{“a”} \mid \text{“x”} \mid \text{“y”} \mid \text{“z”} \\ \langle \textit{method\_call} \rangle &::= \langle \textit{identifier} \rangle\ \text{“(”}\ \langle \textit{name} \rangle\ [\ \text{“,”}\ \langle \textit{name} \rangle\ ]\dots\ \text{“)”} \end{aligned}$$

5. (*7 points*) Suppose that the parse tree at the right is based on a context-free grammar G that has exactly five production rules.



a) Give the five production rules that must be part of G for this parse tree to be valid.

b) What is the yield of this parse tree, that is, the string that is being parsed?

c) Give the left derivation corresponding to this parse tree.

d) Give the right derivation corresponding to this parse tree.

e) Draw a parse tree using the same grammar for the string $abdccd$.