The first test for this course will be given in class on Friday, March 5. (If you cannot be in class in person, you should contact me to arrange accommodations.) The test covers all of the material that we have done up to section 2.4 (with somewhat more emphasis on Chapter 1).

The test will include some "short essay" questions that ask you to define something, or discuss something, or explain something, and so on. Other than that, you can expect most of the questions to be similar to problems that have been given on the homework. You can expect to do a few simple proofs, including at least one formal proof of the validity of an argument and at least one more informal mathematical proof. There might be a simple proof by induction.

Here are some terms and ideas that you should be familiar with for the test:

translations from logic to English, and from English to logic
proposition; propositional logic
the logical operators "and" ($\wedge$), "or" ($\vee$), and "not" ($\neg$)
some information is lost when translating English to logic (for example, "but" vs. "and")
truth table
logical equivalence ($\equiv$)
the conditional or "implies" operator ($\rightarrow$)
definition of $p \rightarrow q$ as $(\neg p) \vee q$
negation of a conditional: $\neg(p \rightarrow q) \equiv p \wedge \neg q$
tautology
Boolean algebra
some basic laws of Boolean algebra (double negation, De Morgan's, commutative, etc.)
logic circuits and logic gates
making a circuit to compute the value of a compound proposition
finding the proposition whose value is computed by a circuit
converse of an implication ($p \rightarrow q$ has converse $q \rightarrow p$)
contrapositive of an implication ($p \rightarrow q$ has contrapositive $(\neg q) \rightarrow (\neg p)$)
an implication is logically equivalent to its contrapositive
predicates; predicate logic
one-place predicate, two-place predicate, etc.
domain of discourse
quantifiers, "for all" ($\forall$) and "there exists" ($\exists$)
negation of a statement that uses quantifiers, such as $\neg\exists x\,(P(x)) \equiv \forall x\,(\neg P(x))$
arguments, valid arguments, and deduction
premises and conclusion of an argument
formal proof of the validity of an argument
how to show that an argument is invalid
translating arguments from English into logic
Modus Ponens, Modus Tollens, and Elimination

mathematical proof

direct proof

existence proof

counterexample

proof by contradiction

rational number (real number that can be expressed as a quotient of integers, $\frac{a}{b}$)

irrational number (real number that is not rational such as π or $\sqrt{2}$)

divisibility, $m \mid n$ (for integers n and m, $m \mid n$ if there is an integer k such that $n = km$)

prime number (greater than 1, and cannot be factored into smaller integers)

proof by mathematical induction

summation notation, for example: $\displaystyle\sum_{k=1}^{n} a_k$

sets

set notations: $\{a, b, c\}$, $\{1, 2, 3, \dots\}$, $\{x \mid P(x)\}$, $\{x \in A \mid P(x)\}$

the empty set, $\varnothing$ or $\{\,\}$

equality of sets: $A = B$ if and only if they contain the same elements

element of a set: $a \in A$

subset: $A \subseteq B$

$A = B$ if and only if both $A \subseteq B$ and $B \subseteq A$

union, intersection, and set difference: $A \cup B$, $A \cap B$, $A \smallsetminus B$

definition of set operations in terms of logical operators

power set of a set: $\mathcal{P}(A)$

universal set

complement of a set (in a universal set): $\overline{A}$

DeMorgan's Laws for sets: $\overline{A \cup B} = \overline{A} \cap \overline{B}$ and $\overline{A \cap B} = \overline{A} \cup \overline{B}$

bitwise operations in Java: `&, |, ~`

correspondence between n-bit binary numbers and subsets of $\{n-1, n-2, \dots, 1, 0\}$

`&`, `|`, and `~` as set operations (intersection, union, complement)

the shift operators `<<` and `>>>`

hexadecimal numbers

ordered pair: (a, b)

cross product of sets: $A \times B$

function $f\colon A \to B$

one-to-one function

onto function

bijective function

B^A, the set of all functions from the set A to the set B

Here are some problems from tests given in previous courses:

*Note that this is **more** than a complete test, more like 1.5 tests!*

1. Define the following terms, as they relate to this course:

a) *logical equivalence* of propositions

b) *subset*

c) *one-to-one function*

2. Let p be the following proposition: "If global temperatures rise by two degrees, then the Greenland ice sheet melts."

a) State the *converse* of p (in natural English)

b) State the *contrapositive* of p (in natural English)

c) State the *negation* of p (in natural English):

3. Use a truth table to show that the proposition $(p \wedge q) \to r$ is logically equivalent to the proposition $p \to (q \to r)$. (Don't forget to say what it is about the truth table that demonstrates the equivalence.)

4. Suppose that $M(x)$ means "x is a math class"; $D(x)$ means "x is difficult"; and $I(x)$ means "x is interesting". The domain of discourse is "all classes." Using only these predicates, translate each of the following statements into predicate logic:

a) There is an interesting math class.

b) All math classes are difficult.

c) All difficult math classes are interesting.

5. Draw a logic circuit that computes the the following logical expression:

$$(A \wedge (B \vee (\neg C))) \wedge (\neg(C \wedge A))$$

6. Simplify the following, so that in the end the $\neg$ operator is applied only to individual predicates.

$$\neg(\exists x \left(P(x) \wedge \forall y \left(Q(x,y) \vee R(x,y)\right)\right)$$

7. Give a *formal proof* that the following argument is valid. Don't forget to give a reason for each step in the proof.

$$\begin{array}{l} (q \wedge (\neg s)) \to p \\ s \to t \\ \neg t \\ q \\ \hline \therefore p \end{array}$$

8. Consider the proposition $\forall x\, \exists y\, L(x,y)$ and the proposition $\exists y\, \forall x\, L(x,y)$. Carefully explain the difference in the meanings of these two propositions. It will be helpful to use some specific predicate L as an example.

9. Give a *proof by contradiction* of the following statement: If x^2 is an odd integer, then x is also odd.

10. Prove that for any integers a, b, and c, if a is divisible by c and b is divisible by c, then the sum $a+b$ is also divisible by c.

11. Let A, B, and C be the sets:

$$A = \{2,3,5,7,11,13\} \qquad B = \{1,2,3\} \qquad C = \{a,b\}$$

Find the following sets:

a) $A \cap B =$ ______________________________

b) $A \smallsetminus B =$ ______________________________

c) $C \times B =$ ______________________________

d) $\mathcal{P}(C) =$ ______________________________

12. Let X be the set $X = \big\{\, a,\ \{b\},\ \{a,b\},\ \{a,\{b\}\},\ \{a,\{a\}\}\,\big\}$.

a) Is the set $\{a\}$ an *element* of X or a *subset* of X or neither or both? Why?

b) Is the set $\{a,b\}$ an *element* of X or a *subset* of X or neither or both? Why?

c) Is the set $\{a,\{b\}\}$ an *element* of X or a *subset* of X or neither or both? Why?

13. Suppose that n and m are variables of type *int* in a Java program, with values $n =$ `0xABCDEF` and $m =$ `0x123456`. What is the value of the following expression? (Explain your answer!)

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(n & 0xFFF000) | (m & 0xFFF)
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14. What is meant by a *universal set*? Where are universal sets used, and why are they necessary. Give an example that uses a universal set.

15. Discuss the principle of mathematical induction. (What does it say? Why is it true, intuitively? What is meant by the base case and the inductive case of an induction?)