1. Define the terms alphabet and language... 
   **Answer:** An alphabet, $\Sigma$, is a finite non-empty set. The elements of $\Sigma$ are considered to be "symbols". A language over $\Sigma$ is a set of strings made up of symbols from $\Sigma$.

2. Let $L = \{a, a^2, a^3\}$. What strings are in the language $L^*$?
   **Answer:** $L^*$ contains the empty string plus all strings of $a$'s and $b$'s that start with an $a$ and do not contain $bbb$ as a substring.

3. Draw a DFA that accepts $L = \{w \in \{0, 1\}^* \mid \text{w has substring } abab\}$.
   **Answer:**
   ![DFA Diagram](image)

4. Suppose $M$ is a DFA over the alphabet $\Sigma$. Explain what it means to say that a string $w \in \Sigma^*$ is accepted by $M$.
   **Answer:** $w$ is accepted by $M$ if when $M$ starts in its start state and reads the entire string, the state in which $M$ finishes is an accepting state. (Formally, if $M = (Q, \Sigma, q_0, \delta, F)$, $w$ is accepted if $\delta^*(q_0, w) \in F$.)

5. Consider the language $\{w \in \{0, 1\}^* \mid w \text{ starts with } a a \text{ and ends with } b b\}$. a) Give a regular expression for the language. b) Draw an NFA that accepts this language.
   a) $aa(a|b)^*bb$
   b) ![NFA Diagram](image)
6. Use the NFA-To-DFA conversion algorithm on this NFA:

7. Find a CFG for \( \{ a^n b^m c^n \mid n, m \in \mathbb{N} \} \) and explain how it works.
   Answer: 
   \[ S \rightarrow aSc \]
   \[ S \rightarrow T \]
   \[ T \rightarrow bT \]
   \[ T \rightarrow E \]
   The first rule makes equal numbers of a's and c's, with all the a's on the left and c's on the right. Then T makes any number of b's, between the a's and c's, using a new symbol, T'. To make the b's ensure they can only be between the a's, c's.

8. Given this CFG, give a left derivation and parse tree for aaaccbba
   
   \[ S \rightarrow ABA \]  Answer: \[ S \rightarrow ABA \]
   \[ A \rightarrow aAc \]  \[ A \rightarrow aAcBA \]
   \[ B \rightarrow Bb \]  \[ B \rightarrow aaccBB \]
   \[ A \rightarrow a \]  \[ A \rightarrow aaccBB \]
   \[ B \rightarrow \]  \[ B \rightarrow aaccBB \]
   \[ A \rightarrow E \]  \[ A \rightarrow aaccBBa \]

9. One of these languages is regular, one is context-free. Give a regular expression for the regular language and a CFG for the context-free.
   a) \( \{ a^n b^m \mid n > m \} \)
   b) \( \{ a^n b^m \mid n > 0 \text{ and } m > 1 \} \)
   c) \( \{ a^n b^m \mid m = n^2 \} \)
   Answer: 
   a) \[ S \rightarrow aSB \] \( a^*bba^* \) (regular)  
   b) \[ S \rightarrow aS \]  \( S \rightarrow E \) (context-free)

10. a) What is nondeterminism? What is non-deterministic about NFA’s?
    b) What about grammars? Are they deterministic or non-deterministic?
    Answer: 
    a) Nondeterminism means that a computation can include steps where the action taken is not fully specified but can be selected at random from several possible actions. An NFA might have several different states it can jump to for a given input, and it can have ε-transitions that it can either follow or not.
    b) Grammars are non-deterministic since they can be several production rules that can be applied to the same non-terminal.