

*This homework is due in class on Wednesday, December 5.*

1. Consider the following context-free grammar:

$$\begin{aligned} S &\longrightarrow aTaRa \\ T &\longrightarrow bTc \\ R &\longrightarrow ccRb \\ T &\longrightarrow d \\ R &\longrightarrow \varepsilon \end{aligned}$$

- a) The string  $abdcaccccbba$  is in the language generated by this grammar. Draw a parse tree for  $abdcaccccbba$ .
- b) What is the shortest string generated by this grammar? Why?
- c) Write down five (5) other strings that are in the language generated by this grammar.
2. For each of the following languages, find a context-free grammar that generates the language. **Briefly explain how each grammar works.**
- a)  $L_1 = \{a^n b^m \mid n, m \in \mathbb{N}, \text{ and } n > m > 0\}$
- b)  $L_2 = \{w \in \{a, b, c\}^* \mid w \text{ contains the substring } abcabc\}$
- c)  $L_3 = \{a^n b a^m b a^{n+m} \mid n, m \in \mathbb{N}\}$
- d)  $L_4 = \{a^n b^m c^n \mid n, m \in \mathbb{N}\}$
- e)  $L_5 = \{a^i b^j c^k d^n \mid i, j, k, n \in \mathbb{N} \text{ and } i + j = k + n\}$
3. Use the pumping lemma for context-free languages to show that the language

$$L = \{a^n b a^n b a^n \mid n \in \mathbb{N}\}$$

is **not** context-free.

4. Use the pumping lemma for context-free languages to show that the language

$$L = \{a^n b^m c^k \mid n, m, k \in \mathbb{N} \text{ and } n \geq m \geq k\}$$

is **not** context-free.

5. For each of the following languages, find a general grammar that generates the language. **Briefly explain how each grammar works.**
- a)  $L = \{a^n b a^n b a^n \mid n \in \mathbb{N}\}$
- b)  $L = \{a^{2^n} \mid n \in \mathbb{N}\}$
- c)  $L = \{ww \mid w \in \{a, b, c\}^*\}$