This is the homework for the week of September 13–17, covering Chapter 1, Sections 6 and 7. It is due in class on Monday, September 15. Remember that you can work with other people in the class, but you should write up your solutions in your own words to turn in. Unsupported answers will not receive much credit!

For the proofs that you are asked to give on this homework, you should give informal, but careful and complete, proofs of the kind that are typically given by mathematicians. You are not required to give formal "valid argument" proofs. Note, by the way, that to "disprove" soemthing means to prove its negation.

There will be a test next week on Wednesday, Septembber 22. It will cover Chapter 1, Sections 1 through 7. Because of the test, the homework for this week is due on Monday instead of Wednesday. Although we will start Section 1.8 on Friday of this week, that section will not be on the test and is not covered on this homework set.

1. Each of the following statements is a potential theorem that is expressed somewhat informally, following the usual mathematical practice. Translate each statement into a statement of formal predicate logic, using quantifiers as appropriate. Introduce predicates and variables as necessary. You are **not** being asked to prove the statements in this problem, only to translate them into logic.

a) If a and b are odd integers, then ab is also an odd integer.

b) The product of two integers is odd if and only if both of the integers are odd.

c) The product of any two irrational numbers is irrational.

d) Not every prime number is odd.

e) Suppose that p is a prime number and that n and m are integers such that the product nm is divisible by p. Then either n is divisible by p or m is divisible by p.

2. Suppose that n, m, and k are integers and that n is divisible by k. Show that the product nm is also divisible by k.

3. Prove: If x is any real number, then either x or $\pi - x$ is irrational. [Note: π is known to be an irrational number.]

4. Prove or disprove:

a) If x is an irrational number, then x^2 is also irrational.

b) If x^2 is an irrational number, then x is also irrational. (Hint: Consider the contrapositive.)

5. Prove or disprove:

a) For any integer n, if n is divisble by 4, then n^2 is divisible by 4.

b) For any integer n, if n^2 is divisble by 4, then n is divisible by 4.

6. Prove using a proof by contradiction: Suppose that x, y, and z are real numbers such that $x + y + z > 30$. Then at least one of x, y, and z is greater than 10.