This homework is due on Section 2.2 is due at the start of Lab 2 on Tuesday, January 28. For this homework only, you should write your answers on this sheet and turn it in, and it is not necessary to justify your answers unless the question specifically asks you to do so. Don’t forget to put your name at the top!

1. This exercise is about limits and also about understanding conventions for drawing and reading graphs. The picture shows the graph of a function, \( y = f(x) \). Fill in all the values below the table. If the value does not exist because it is infinite, write \(+\infty\) or \(-\infty\), as appropriate. If the value does not exist for some other reason, write DNE. Some answers might be approximations.

\[
\begin{align*}
f(-3) &= \_\_\_ \quad \lim_{x \to -3^+} f(x) &= \_\_\_ \quad \lim_{x \to -3^-} f(x) &= \_\_\_ \quad \lim_{x \to -3} f(x) &= \_\_\_ \\
f(-2) &= \_\_\_ \quad \lim_{x \to -2^+} f(x) &= \_\_\_ \quad \lim_{x \to -2^-} f(x) &= \_\_\_ \quad \lim_{x \to -2} f(x) &= \_\_\_ \\
f(-1) &= \_\_\_ \quad \lim_{x \to -1^+} f(x) &= \_\_\_ \quad \lim_{x \to -1^-} f(x) &= \_\_\_ \quad \lim_{x \to -1} f(x) &= \_\_\_ \\
f(0) &= \_\_\_ \quad \lim_{x \to 0^+} f(x) &= \_\_\_ \quad \lim_{x \to 0^-} f(x) &= \_\_\_ \quad \lim_{x \to 0} f(x) &= \_\_\_ \\
f(1) &= \_\_\_ \quad \lim_{x \to 1^+} f(x) &= \_\_\_ \quad \lim_{x \to 1^-} f(x) &= \_\_\_ \quad \lim_{x \to 1} f(x) &= \_\_\_ \\
f(2) &= \_\_\_ \quad \lim_{x \to 2^+} f(x) &= \_\_\_ \quad \lim_{x \to 2^-} f(x) &= \_\_\_ \quad \lim_{x \to 2} f(x) &= \_\_\_ \\
f(3) &= \_\_\_ \quad \lim_{x \to 3^+} f(x) &= \_\_\_ \quad \lim_{x \to 3^-} f(x) &= \_\_\_ \quad \lim_{x \to 3} f(x) &= \_\_\_ \\
f(4) &= \_\_\_ \quad \lim_{x \to 4^+} f(x) &= \_\_\_ \quad \lim_{x \to 4^-} f(x) &= \_\_\_ \quad \lim_{x \to 4} f(x) &= \_\_\_
\end{align*}
\]

2. Fill in the following values, similarly to problem 1, if \( g(x) = \begin{cases} 2x & \text{if } -1 < x \leq 1 \\ x^2 - 1 & \text{if } x \leq -1 \\ 3 - x^2 & \text{if } x > 1 \end{cases} \)

\[
\begin{align*}
g(1) &= \_\_\_ \quad \lim_{x \to 1^+} g(x) &= \_\_\_ \quad \lim_{x \to 1^-} g(x) &= \_\_\_ \quad \lim_{x \to 1} g(x) &= \_\_\_ \\
g(-1) &= \_\_\_ \quad \lim_{x \to -1^+} g(x) &= \_\_\_ \quad \lim_{x \to -1^-} g(x) &= \_\_\_ \quad \lim_{x \to -1} g(x) &= \_\_\_
\end{align*}
\]
3. Fill in the following table of values of the function \( f(x) = \frac{1 - \cos(x)}{x} \) near \( x = 0 \), using a calculator to compute the values. You can round your answers to 6 decimal places.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( \frac{1 - \cos(x)}{x} )</th>
<th>( x )</th>
<th>( \frac{1 - \cos(x)}{x} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.1</td>
<td></td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>-0.01</td>
<td></td>
<td>0.01</td>
<td></td>
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<tr>
<td>-0.001</td>
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<td>-0.0001</td>
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<tr>
<td>-0.00001</td>
<td></td>
<td>0.00001</td>
<td></td>
</tr>
</tbody>
</table>

Based on the table, what do you think is the value of \( \lim_{x \to 0} \frac{1 - \cos(x)}{x} \)? Why?

4. For each part of this problem, sketch a graph of a function \( f(x) \) that has the specified propeties. For all values of \( x \) except for \(-2, 0, \) and \(2\), the function should be “nice” (that is, for \( x \neq 2 \), the limit exists at \( x \) and is equal to the value of \( f(x) \)).

\[ a) \] For the coordinate grid on the left below: \( \lim_{x \to 2} f(x) = 1.5 \), \( f(2) = -1 \), \( \lim_{x \to 0^+} f(x) = 2 \), \( \lim_{x \to 0^-} f(x) = -2 \), \( f(0) = 1 \), \( \lim_{x \to -2} f(x) = 1 \), \( \lim_{x \to 0^+} f(x) = 1 \), and \( f(-2) = 2 \).

\[ b) \] For the coordinate grid on the right below: \( \lim_{x \to -2} f(x) = +\infty \), \( \lim_{x \to 2} f(x) = -\infty \), \( \lim_{x \to 0^-} f(x) = -\infty \), and \( \lim_{x \to 0^+} f(x) = +\infty \).