

This homework on Sections 2.3 and 2.4 is due at the start of class on Wednesday, February 5. Write your answers on separate paper. Remember that you can work on homework with other students in the class, but you should write up your own solutions in your own words. Don't forget to show your work!

1. The following limit could be done simply by “plugging in,” but for this problem, you should write out the process of applying one limit law at a time to compute the limit:

$$\lim_{x \rightarrow 4} \frac{x^2 - 3x}{x + \sqrt{x}}$$

2. Compute the following limits. You do not have to apply the limit laws individually. Just do some algebra, if necessary, and use “plugging in.”

a) $\lim_{x \rightarrow -2} \frac{x^2 + 5x + 6}{x + 2}$

b) $\lim_{x \rightarrow -2} \frac{x^2 + 5x + 4}{x + 2}$

c) $\lim_{x \rightarrow -2} \frac{x + 2}{x^2 + 5x + 4}$

d) $\lim_{t \rightarrow 3} \frac{t^2 - 9}{t^2 - 2t - 3}$

e) $\lim_{t \rightarrow 3} \frac{\sqrt{t+1} + 4}{5 - t}$

f) $\lim_{t \rightarrow 3} \frac{\sqrt{t+1} - 2}{t - 3}$

3. Use the Squeeze Theorem to prove that $\lim_{x \rightarrow 0} x^2 \sin\left(\frac{1}{x}\right)$ equals zero.

4. For each of the following functions, find all values of x where the function is discontinuous. In each case, determine whether the discontinuity is a removable discontinuity, a jump discontinuity, or an infinite discontinuity.

a) $f(x) = \frac{x^2 - 1}{(x + 1)(x - 2)}$

b) $g(x) = \frac{(x + 2)(x + 3)}{(x - 2)(x^2 - 9)}$

5. Use the Intermediate Value Theorem to prove the following statements.

a) The polynomial $p(x) = x^3 + 2x - 1$ has a root somewhere in the interval $[0, 1]$.

b) The equation $x^3 - 5x^2 + 2x = -1$ has a solution in the interval $[-1, 5]$.

c) The equation $\cos(x) = x$ has a solution satisfying $0 < x < \frac{\pi}{2}$. (Remember that angles are measured in radians, not degrees.)