Math 130-01, Spring 2020
Sample Answers to Lab #2

1. a) \( \lim_{x \to 2} \frac{x^2 - x - 2}{x^2 + x - 6} \)
   
   \[
   = \lim_{x \to 2} \frac{(x-2)(x+3)}{(x-2)(x+3)} \quad \text{algebra}
   
   = \lim_{x \to 2} \frac{x+3}{x+3} \quad \text{algebra}
   
   = \lim_{x \to 2} \frac{(x+1)}{x+1} \quad \text{quotient law}
   
   = \lim_{x \to 2} \frac{x+2}{x+3} \quad \text{sum law}
   
   = \frac{2+1}{2+3} \quad \text{basic laws}
   
   = \frac{3}{5} \quad \text{arithmetic}
   
   \]

b) \( \lim_{x \to -1} \frac{x^2 + 2x + 1}{x^2 + x + 1} \)

\[
= \lim_{x \to -1} \frac{(x+1)^2}{x(x+1)} \quad \text{algebra}

= \lim_{x \to -1} \frac{x+1}{x} \quad \text{algebra}

= \lim_{x \to -1} \frac{(x+1)}{x} \quad \text{quotient law}

= \lim_{x \to -1} \frac{x+2}{x+3} \quad \text{sum law}

= \frac{-1+1}{-1} \quad \text{basic laws}

= \frac{0}{-1} = 0 \quad \text{arithmetic}

\]

c) \( \lim_{x \to 1} \left( \frac{x-1}{3x^2 - 4x + 1} \right)^3 \)

\[
= \left( \lim_{x \to 1} \frac{x-1}{3x^2 - 4x + 1} \right)^3 \quad \text{polynomial law}

= \left( \lim_{x \to 1} \frac{x-1}{(x-1)(3x-1)} \right)^3 \quad \text{algebra}

= \left( \lim_{x \to 1} \frac{1}{3x-1} \right)^3 \quad \text{algebra}

= \left( \lim_{x \to 1} \frac{1}{3x-1} \right)^3 \quad \text{quotient law}

= \left( \lim_{x \to 1} \frac{1}{3x-1} \right)^3 \quad \text{basic laws}

= \frac{1}{8} \quad \text{arithmetic}

\]
2. a) \[
\lim_{x \to 3} (f(x)^2 + 3g(x))
\]
\[
= \left( \lim_{x \to 3} f(x)^2 \right) + \left( \lim_{x \to 3} 3g(x) \right) \quad \text{Sum Law}
\]
\[
= \left( \lim_{x \to 3} f(x) \right)^2 + \left( \lim_{x \to 3} 3g(x) \right) \quad \text{Power Law and Product Law}
\]
\[
= 5^2 + 3 \cdot (-2)
\]
\[
= 25 - 6 = 19 \quad \text{given for \( \lim_{x \to 3} f(x) \) and \( \lim_{x \to 3} g(x) \)}
\]

b) \[
\lim_{x \to 3} \frac{h(x)}{f(x) + g(x)} = \frac{\lim_{x \to 3} h(x)}{\lim_{x \to 3} [f(x) + g(x)]} \quad \text{Quotient Law}
\]
\[
= \frac{\lim_{x \to 3} h(x)}{\left( \lim_{x \to 3} f(x) + \lim_{x \to 3} g(x) \right)} \quad \text{Sum Law}
\]
\[
= \frac{4}{5 + (-2)}
\]
\[
= \frac{4}{3} \quad \text{arithmetic}
\]

c) \[
\lim_{x \to 3} (x f(x) + x^2 h(x)) = \left( \lim_{x \to 3} x f(x) \right) + \left( \lim_{x \to 3} x^2 h(x) \right) \quad \text{Sum Law}
\]
\[
= \left( \lim_{x \to 3} x \right) \left( \lim_{x \to 3} f(x) \right) + \left( \lim_{x \to 3} x^2 \right) \left( \lim_{x \to 3} h(x) \right) \quad \text{Product Law}
\]
\[
= \left( \lim_{x \to 3} x \right) \left( \lim_{x \to 3} f(x) \right) + \left( \lim_{x \to 3} x^2 \right) \left( \lim_{x \to 3} h(x) \right) \quad \text{Power Law}
\]
\[
= 3 \cdot 5 + 3^2 \cdot 4 \quad \text{basic laws and given values}
\]
\[
= 15 + 36 = 51 \quad \text{arithmetic}
\]
3) \( \lim_{x \to 0} (f(x) + g(x)) = +\infty \). As \( x \to 0 \), the value of \( f(x) \) gets larger and larger in a positive direction. Adding a number close to 0 to a big positive number is still a big positive number. So \( f(x) + g(x) \to +\infty \).

b) \( \lim_{x \to 0} (f(x) + g(x)) = +\infty \), because adding two big positive numbers gives an even bigger positive number.

c) \( \lim_{x \to 0} \left( \frac{1}{x^2} + \frac{1}{x^2} \right) = \lim_{x \to 0} \frac{-1}{x^2} = -\infty \)

\( \lim_{x \to 0} \left( \frac{3}{x^2} - \frac{1}{x^2} \right) = \lim_{x \to 0} \frac{1}{x^2} = \infty \)

\( \lim_{x \to 0} \left( (17 + \frac{1}{x^2}) - (\frac{1}{x^2}) \right) = \lim_{x \to 0} 17 = 17 \)

(And we could replace 17 with any other number.)

All of these limits are of the form "(\( +\infty \)) + (\( -\infty \))", and what we see is that \( \lim_{x \to 0} f(x) + g(x) \) could be literally anything. So there is no simple rule that applies to limits of the form \( +\infty + (-\infty) \).

4) 9)

Alternative approach: \( y = ax + b \) is a line that must pass through \((-1, 1)\) and \((1, 3)\), so slope is \( a = \frac{3}{2} \) and intercept is \( b = 1 \).
\[ b(x) = 2x^2 \]

\[ f(x) = \frac{1}{x} \]

6) \[
\lim_{x \to -2^+} f(x) = \lim_{x \to -2^+} x + 1 = -1
\]

\[ f(x) = \lim_{x \to -2^-} x^2 = 4a 
\]

So we need \( 4a = -1 \) \( \Rightarrow a = -\frac{1}{4} \)

\[ f(x) = \lim_{x \to 3^-} x + 1 = 4 
\]

\[ f(x) = \lim_{x \to 3^+} 6a^2 = 9b \]

So we need \( 9b = 4 \) \( \Rightarrow b = \frac{4}{9} \)

5) a) As \( x \) gets bigger and bigger, \( \frac{1}{x} \) gets smaller and smaller, so \( \lim_{x \to +\infty} \frac{1}{x} = 0 \).

b) \( \lim_{x \to +\infty} \left( \frac{x+1}{x} \right) = \lim_{x \to +\infty} \left( 1 + \frac{1}{x} \right) = 1 \), because \( \frac{1}{x} \to 0 \).

(Or, when \( x \) is very large \( x \approx x+1 \), so \( \frac{x+1}{x} \) is close to 1.)

c) \[
\begin{array}{c|c|c|c|c|c}
\hline
x & 10 & 100 & 1000 & 10000 & 100000 \\
\hline
\sqrt{x^2 + x} - x & .48808 & .49876 & .499875 & .4999875 & .49999875 \\
\hline
\end{array}
\]

So it looks like \( \lim_{x \to \infty} \left( \sqrt{x^2 + x} - x \right) = \frac{1}{2} \)

6) The two suggested estimates give the values 1.6 and 6.4. One idea is simply to take the average, giving 4.0 as the estimate. Another idea is to draw a careful graph and draw a tangent line at (1.5, 5.6). The slope of that line represents the velocity at time 1.5. When I did this, my slope was about 3.2 .

7) If the object moves at constant velocity 1.2 \( \text{ft/sec} \) for time 0.3, it travels a distance of \( 1.2 \times 0.3 = 0.36 \) ft, giving position \( 8.7 + 0.36 \) or \( 9.06 \). Lacking more information, this is the best estimate.