1. a) \( p\left(\frac{1}{2}\right) = 3\cdot\left(\frac{1}{2}\right)^5 - \frac{1}{2} - 1 = \frac{3}{32} - \frac{16}{32} - \frac{32}{32} = -\frac{45}{32} \). Since \( p\left(\frac{1}{2}\right) < 0 \) and \( p(1) > 0 \), the IVT says there is a root in the interval \( \left[\frac{1}{2}, 1\right] \).

b) To narrow down to an interval of length \( \frac{1}{4} \), look at \( p\left(\frac{3}{4}\right) \). It will be either positive or negative. If \( p\left(\frac{3}{4}\right) > 0 \), the root will be in \( \left[\frac{3}{4}, 1\right] \); because \( p\left(\frac{3}{4}\right) \) is negative, \( \frac{p\left(\frac{3}{4}\right)}{p(1)} > 0 \). If \( p\left(\frac{3}{4}\right) \) happens to be zero exactly, then we know that the root is \( 0 \) exactly. In fact, by calculator, \( p\left(\frac{3}{4}\right) = -1.038... \). Since \( p\left(\frac{3}{4}\right) < 0 \) and \( p(1) > 0 \), the root is in \( \left[\frac{3}{4}, 1\right] \).

c) Look at \( x = \frac{7}{8} \), which is halfway between \( \frac{3}{4} \) and \( 1 \).
\( p\left(\frac{7}{8}\right) = -0.036... \). Since \( p\left(\frac{7}{8}\right) < 0 \) and \( p\left(\frac{3}{4}\right) > 0 \), the root is in the interval \( \left[\frac{7}{8}, 1\right] \). On the next step, we would find it is in the range \( \left[\frac{7}{8}, \frac{15}{16}\right] \), since \( p\left(\frac{15}{16}\right) > 0 \) and \( p\left(\frac{7}{8}\right) < 0 \).

d) Start with \( a = 0 \), \( b = 1 \). On each step, choose the number \( c = \frac{a+b}{2} \), that is halfway between \( a \) and \( b \).
If \( p(c) \) has the same sign as \( p(a) \), then the root is in \( \left[c, b\right] \); so replace \( a \) with \( c \); if \( p(c) \) has the same sign as \( p(b) \), then the root is in \( \left[a, c\right] \), so replace \( b \) with \( c \). If the new \( \left[a, b\right] \) has a length that is smaller than the desired accuracy, stop. Otherwise continue on to the next step.

This works because at every step, the root is known to be in \( \left[a, b\right] \), and when we stop we can take \( \frac{a+b}{2} \) to be the approximation that we want.
(2) a) Let $D$ be the distance from the car to the lake. Then $f(0) = 0$, $f(2) = D$, $g(0) = D$, $g(2) = 0$.

b) $h(0) = f(0) - g(0) = 0 - D = -D$
$\quad h(2) = f(2) - g(2) = D - 0 = D$

c) By the IVT, because $0$ is between $-D$ and $D$, there must be a $c$ in $[0, 2]$ such that $h(c) = 0$. Then $f(c) = g(c)$, which means you are the same distance from the car at $c$ hours after 7:00 Friday as at $c$ hours after 7:00 on Sunday.

(2) $f(x) = \frac{|x^2 - 9|}{x^2 + x - 12} = \frac{|x^2 - 9|}{(x+4)(x-3)}$. The only points of discontinuity are $x = 3$ and $x = 4$, where the denominator is 0. At $x = 4$, $\frac{|x^2 - 9|}{(x+4)(x-3)}$ had the form $\frac{5}{0}$, so the limit of $f(x)$ from the left or right as $x \to 4$ is infinite. $f(x)$ has an infinite discontinuity at $x = 4$. At $x = 3$, we need to examine the limits from the left and from the right. Note that $f(x) = \frac{|x-3||x+3|}{(x+4)(x-3)}$

$= \frac{|x+3|}{x+4} \cdot \frac{|x-3|}{x-3}$. $\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} \frac{|x+3|}{x+4} \cdot \frac{-3}{x-3} = \frac{6}{7} \cdot (-1) = -\frac{6}{7}$
because $|x-3| = -(x-3)$ and $|x-3| = -(x-3)$. And

$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} \frac{|x+3|}{x+4} \cdot \frac{x-3}{x-3} = \frac{6}{7} \cdot 1 = \frac{6}{7}$. Since for

$x > 3$, $x - 3 > 0$ and $|x-3| = x-3$. Since limits from left and right are different, it's a jump discontinuity.
4. a) Let \( \varepsilon > 0 \). Let \( \delta = \frac{\varepsilon}{5} \). We then have that if
\[ 0 < |x - 5| < \delta, \text{ Then } |f(x) - L| = \left| \frac{5(x - 5) - 15}{x - 10} \right| = \frac{5}{5} \left| x - 5 \right| < 5 \cdot \frac{\varepsilon}{5} = \varepsilon. \]

b) Let \( \varepsilon > 0 \). Let \( \delta = \frac{\varepsilon}{3} \). We then have that if
\[ 0 < |x - 2| < \delta, \text{ Then } |f(x) - L| = \left| \frac{1}{3(x + 1)} - (-5) \right| \]
\[ = \left| -3x + 6 \right| = \left| (3)(x-2) \right| = 13 \cdot |x-2| < 13 \cdot \frac{\varepsilon}{3} = \varepsilon. \]

5. a) \[ \lim_{x \to 2} \frac{\sqrt{x} - \sqrt{2}}{x^2 - 2x} = \lim_{x \to 2} \frac{\sqrt{x} - \sqrt{2}}{x^2 - 2x} \cdot \frac{\sqrt{x} + \sqrt{2}}{\sqrt{x} + \sqrt{2}} \]
\[ = \lim_{x \to 2} \frac{x - 2}{x(x-2)(\sqrt{x} + \sqrt{2})} = \lim_{x \to 2} \frac{1}{x(\sqrt{x} + \sqrt{2})} = \frac{1}{2(\sqrt{2})} = \frac{1}{2\sqrt{2}} \]

b) \[ \lim_{x \to 7} \frac{\sqrt{x+2} - 3}{\sqrt{x-2} - 2} = \lim_{x \to 7} \frac{\sqrt{x+2} - 3}{\sqrt{x-2} - 2} \cdot \frac{\sqrt{x+2} + 3}{\sqrt{x+2} + 3} \]
\[ = \lim_{x \to 7} \frac{(\sqrt{x+2} - 3)(\sqrt{x+2} + 3)}{(\sqrt{x-2} - 2)(\sqrt{x-2} + 2)} = \lim_{x \to 7} \frac{(x+2-9)}{(x-2+4)} = \frac{x-7}{x-2} = \frac{2+2}{3+3} = \frac{2}{3} \]

c) \[ \lim_{x \to 1} \left( \frac{2}{(x-3)(x-1)} - \frac{1}{(x-2)(x-1)} \right) \]
\[ = \lim_{x \to 1} \left( \frac{2(x-2) - 1(x-3)}{(x-3)(x-1)(x-2)} \right) = \lim_{x \to 1} \left( \frac{2x - 4 - x + 3}{(x-3)(x-1)(x-2)} \right) \]
\[ = \lim_{x \to 1} \left( \frac{x-1}{(x-3)(x-1)(x-2)} \right) = \lim_{x \to 1} \frac{1}{(x-3)(x-2)} \]
\[ = \frac{1}{(1-3)(1-2)} = \frac{1}{(2)(1)} = \frac{1}{2} \]