Calculus II, Spring 2005
Lab 2

Math 131-01
February 15

Exercises to hand in: *Please hand in your responses to the following exercises. You should work on these problems in a group and turn in a single solution for the entire group. If you have extra time, you can start on this week's homework.*

1. Let k be a positive constant. Show that for any constants A and B, the function defined by $y = A\sin(\sqrt{k}\,x) + B\cos(\sqrt{k}\,x)$ is a solution of the differential equation $y'' = -ky$

2. Suppose that an object is dropped, with an initial velocity of zero, near the surface of the Earth. Let y be the number of feet it has fallen, as a function of time, t, measured in seconds. In a vacuum, where there is no air resistance, the object has an acceleration of 32 (feet per second per second), so that y satisfies the differential equation $y'' = 32$. (The acceleration is positive since we are measuring distance traveled downwards.) This differential equation has the general solution $y'' = 16t^2 + C_1t + C_2$, where C_1 and C_2 are constants. In this case, since $y'(0) = 0$ and $y(0) = 0$, both constants are equal to zero, and so the distance fallen in t seconds is given by $y = 16t^2$. But this only applies when there is no air resistance. Air resistance adds an additional force that is proportional to the velocity of the object (or at least approximately so for most objects). The faster the object moves, the more air resistance. With air resistance taken into account, the distance fallen by the object satisfies the differential equation $y'' = 32 - ky'$, where k is a positive constant. You don't have the tools needed to solve this equation, so I will tell you the solution.

a) Why is there a minus sign rather than a plus sign in the equation $y'' = 32 - ky'$?

b) Show that $y = \frac{32}{k}t + \frac{32}{k^2}(e^{-kt} - 1)$ is a solution of the differential equation $y'' = 32 - ky'$ with initial conditions $y(0) = y'(0) = 0$.

c) Describe in words what happens in the long run to the acceleration, velocity, and distance fallen, based on the formulas for these quantities. Explain why this behavior makes sense physically.

d) The value of the constant k depends on the particular object under discussion. Based on the information in this problem, explain why an ant dropped from a great height will be uninjured, while a fall from the same height would kill a person.

3. After 10 years, 0.021% of a certain radioactive isotope has decayed. How long will it be until 99% is gone?

4. Exercise number 77 on page 381 of the textbook is a true/false question: "If prices are rising at a rate of 0.5% per month, they they are rising at a rate of 6% per year." The answer to this question is *false*. Carefully explain why the statement is false, and determine the correct rate of increase per year.