Calculus II, Spring 2005 — Math 131-01
Lab 9 — March 22

1. Let M be a number greater than 1. (We will let $M \to \infty$, so you should think of it as being large.) Consider the function $y = \frac{1}{x}$ on the interval $1 \leq x \leq M$. When this curve is rotated about the x-axis, a surface of revolution is generated that looks something like the horn on the trumpet. When the region under the curve is rotated, the corresponding solid of revolution is generated.

a) Find the volume of the solid that is generated when the region under the curve $y = \frac{1}{x}$ on the interval $1 \leq x \leq M$ is rotated about the x-axis, and determine what happens to the volume as $M \to \infty$.

b) Find the area under the curve $y = \frac{1}{x}$ on the interval $1 \leq x \leq M$, and determine what happens to the area as $M \to \infty$.

c) Write down the integral that gives the surface area of the surface of revolution that that is generated when the curve $y = \frac{1}{x}$ on the interval $1 \leq x \leq M$ is rotated about the x-axis. You can't compute this integral, but recall that if $f(x) > g(x)$ on an interval $[a, b]$, then $\int_a^b f(x)\,dx > \int_a^b g(x)\,dx$. Use this fact and your answer to part **b)** to determine what happens to the surface area as $M \to \infty$.

d) Consider the infinite trumpet-shaped surface that is formed by taking $1 \leq x < \infty$. This is a famous mathematical example that is sometimes referred to as "Gabriel's Horn." Your answers to the previous parts of this exercise should have shown that Gabriel's Horn has an infinite surface area, but that the inside of the horn is a finite volume. This might seem like a paradox. To see why, think about painting the horn. Since the surface area is infinite, it's impossible to paint it with a finite volume of paint. But the volume inside the horn is finite, so it can be filled with a finite volume of paint. But if you fill it with paint, you've covered the surface with paint. So, haven't you painted it? Think about this seeming paradox and try to explain why it's not really a paradox. (Hint: If you can't figure it out, it's not because you don't understand mathematics—it's because you don't understand paint.) If you can't give a definitive answer, discuss the paradox and your reaction to it.

2. We will be spending the next few weeks learning about techniques of integration. So far, the only techniques that you know are basic formulas, algebraic manipulation, and substitution. Even with only these methods, it is sometimes difficult to know how to approach a given integral. Among the following twelve integrals, eight can be done using techniques that you already know. Identify the eight do-able integrals, and do them.

a) $\displaystyle\int x^3 \tan(x^4)\,dx$ **b)** $\displaystyle\int \frac{x^2}{x-1}\,dx$ **c)** $\displaystyle\int \frac{\sin^2(x^2) + \cos^2(x^2)}{x^2}\,dx$

d) $\displaystyle\int x e^{x^2} \sin(2e^{x^2})\,dx$ **e)** $\displaystyle\int \frac{x-1}{x^2}\,dx$ **f)** $\displaystyle\int x(\sin^2(x^2) - \cos^2(x^2))\,dx$

g) $\displaystyle\int x^2 e^{x^2}\,dx$ **h)** $\displaystyle\int \frac{3x^2-1}{x^3-x}\,dx$ **i)** $\displaystyle\int x\sqrt[3]{1-x}\,dx$

j) $\displaystyle\int \sin(x) e^{3\cos(x)}\,dx$ **k)** $\displaystyle\int \frac{1}{x^3-x}\,dx$ **l)** $\displaystyle\int x\sqrt{1+x^4}\,dx$