The final exam for this test is scheduled for 8:30 AM on Monday, May 9, in our regular classroom. The exam ends officially at 11:30, but most people will not need the full two hours. You should expect it to be about twice as long as one of the in-class tests. It will be 7 or 8 pages long.

The exam is cumulative, with some emphasis on what we have covered since the third test. About half of the final exam will be on this recent material. The other half will be divided among topics from the first three tests. Since the test, we have been working on infinite series, including power series and Taylor series. The reading is from Sections 8.3 to 8.5 and 9.1 to 9.3, but we have not covered everything in those sections.

I will hold office hours as follows in the days leading up to the exam:

- Tuesday, May 3: 1:00 PM – 3:00 PM
- Wednesday, May 4: 11:00 AM – 3:00 PM
- Friday, May 6: 12:00 Noon – 4:00 PM
- Sunday, May 8: 12:00 Noon – 3:00 PM

You will not need a calculator for the exam. A basic non-graphing calculator will be provided to you, and you will be permitted to use only the calculator that is provided. Scrap paper will also be provided. All you need is a pencil or pen.

Here are some terms and ideas covered since the third in-class test:

infinite series, $\displaystyle\sum_{k=1}^{\infty} a_k$

partial sum of an infinite series

convergence and divergence of infinite series

convergence of a series means the convergence of the sequence of partial sums

the sum of an infinite series

harmonic series, $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$

the harmonic series is divergent (but very slowly!)

Geometric Series: $\displaystyle\sum_{k=0}^{\infty} ar^k = a + ar + ar^2 + ar^3 + \cdots$ (where a and r are numbers)

a geometric series converges when $|r| < 1$, and the sum is $\displaystyle\sum_{k=0}^{\infty} ar^k = \frac{a}{1-r}$

Integral test: If $f(x)$ is a decreasing, positive function of x for $x \geq 1$, then the series $\sum_{k=1}^{\infty} f(k)$ converges if and only if the improper integral $\int_1^{\infty} f(x)\,dx$ converges.

p**-series**: If p is a constant, the series $\sum_{k=1}^{\infty} \frac{1}{k^p}$ converges if $p > 1$ and diverges if $p \leq 1$.

Divergence test: If $\lim_{k\to\infty} a_k \neq 0$, then the series $\sum_{k=0}^{\infty} a_k$ diverges. (If the limit of the sequence is zero, you get no information about the convergence of the series.)

Ratio test: Suppose $a_k > 0$ for all k. Let $L = \lim_{k\to\infty} \frac{a_{k+1}}{a_k}$. If $L < 1$, then the series $\sum_{k=1}^{\infty} a_k$ converges. If $L > 1$, then the series $\sum_{k=1}^{\infty} a_k$ diverges. (If $L = 1$, you get no information about the convergence of the series.)

Comparison test: Suppose that $0 \leq a_k \leq b_k$ for all k (or for all large k). If $\sum_{k=1}^{\infty} b_k$ converges, then so does $\sum_{k=1}^{\infty} b_k$; if $\sum_{k=1}^{\infty} a_k$ diverges, then so does $\sum_{k=1}^{\infty} a_k$.

Alternating Series Test: If $a_k > 0$ for all k and $a_1 \geq a_2 \geq a_3 \geq a_4 \geq \cdots$ and $\lim_{k\to\infty} a_k = 0$, then the alternating series $\sum_{k=0}^{\infty} (-1)^k a_k$ (that is, $a_o - a_1 + a_2 - a_3 + \cdots$) converges.

absolute convergence

conditional convergence

absolute convergence implies conditional convergence

power series $\sum_{k=0}^{\infty} c_k (x-a)^k = c_o + c_1(x-a) + c_2(x-a)^2 + c_3(x-a)^3 + \cdots$

radius of convergence of a power series

interval of convergence of a power series

term-by-term differentiation of power series

term-by-term integration of power series

If a function $f(x)$ is equal to a power series, $f(x) = \sum_{n=0}^{\infty} c_n (x-a)^n$, on some open interval containing a, then the coefficients in the power series are given by $c_n = \frac{f^{(n)}(a)}{n!}$.

Taylor series of a function $f(x)$ at a: $\sum_{n=0}^{\infty} \frac{f^{(n)}}{n!}(x-a)^n$. (Might not be equal to $f(x)$!)

Taylor polynomial (just a Taylor series cut off after a finite number of terms)

Here are some of the most important ideas from earlier in the term:

Riemann sum

definite integral

integrable function; every continuous function is integrable

relation of definite integral, Riemann sum, and area under a curve

the Fundamental Theorem of Calculus

the average value of a function on an interval

Mean Value Theorem for Integrals

the method of substitution for evaluating integrals

displacement, velocity, and acceleration

the general "future value" formula: $f(t) = f(a) + \int_a^t f'(x)\,dx$

areas of regions between curves

volume as the integral of cross-sectional area

volume of a solid of revolution, using the disk or washer method

volume of a solid of revolution, using the shell method

arc length

linear density

work and force

differential equations, separation of variables, and initial value problems

exponential growth and decay

integration by parts

partial fractions (for linear factors only) integrals of the form $\int \sin^n(x)\cos^m(x)\,dx$

basic trigonometric substitution, $x = a\sin(\theta)$ and $x = a\tan(\theta)$

improper integrals

infinite sequences

the limit of an infinite sequence, $\lim_{n\to\infty} a^n$

a non-decreasing sequence that is bounded above must converge, to a number $\leq$ the upper bound

geometric sequence $\{r^n\}_{n=1}^{\infty}$

See also the review sheets for the first three tests, which are available on-line.

Here are a few suggested problems from Chapter 9:

- Section 9.1: # 17, 29
- Section 9.2: # 9, 11, 21, 23, 33, 37
- Section 9.3: # 13, 19