Section 5.1

18. The left Riemann sum with \( n = 4 \) is 
\[
\sum_{i=1}^{n} f(x_i^*) \Delta x
\]
where \( \Delta x = \frac{5-1}{4} = 1 \) and \( x_i^* \) is the left endpoint of the \( i \)th subinterval. Here, we see that \( x_i^* = i \). So the sum becomes 
\[
f(1) \cdot 1 + f(2) \cdot 1 + f(3) \cdot 1 + f(4) \cdot 1.
\]
Since \( f(x) = \frac{1}{x} \), the sum is 
\[
1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}.
\]
The right Riemann sum is similar, except that the values of \( x \) at the right endpoints are 2, 3, 4, 5, so the sum is 
\[
\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}.
\]

38. For \( n = 2 \), \( \Delta x = 2 \), the intervals are [0, 2] and [2, 4], and the midpoints are \( x_1^* = 1 \), \( x_2^* = 3 \). The Riemann sum is 
\[
V(x_i^*) \Delta x = V(1) \cdot 2 + V(3) \cdot 2
\]
\[
= 30 + 35 + 2
\]
\[
= 55 + 35 + 2
\]
\[
= 90 + 2
\]
\[
= 92
\]

For \( n = 4 \), \( \Delta x = 1 \), the midpoints are 0.5, 1.5, 2.5 and 3.5 and the sum is 
\[
V(0.5) \cdot 1 + V(1.5) \cdot 1 + V(2.5) \cdot 1 + V(3.5) \cdot 1
\]
\[
= 25 + 35 + 30 + 40
\]

40. a) \( 1 + 3 + 5 + 7 + \ldots + 99 = \sum_{n=1}^{50} (2n-1) \), because 
\[
2 \cdot 1 - 1 = 1, \ 2 \cdot 2 - 1 = 3, \ 2 \cdot 3 - 1 = 5, \ldots \ \ 2 \cdot 50 - 1 = 99.
\]

b) \( 4 + 9 + 14 + \ldots + 44 = \sum_{k=0}^{8} 5k + 4 \), because 
\[
5 \cdot 0 + 4 = 4, \ 5 \cdot 1 + 4 = 9, \ 5 \cdot 2 + 4 = 14, \ldots, \ 5 \cdot 8 + 4 = 44
\]

c) \( 3 + 8 + 13 + \ldots + 63 = \sum_{j=1}^{13} 5j - 2 \), because 
\[
5 \cdot 1 - 2 = 3, \ 5 \cdot 2 - 2 = 8, \ 5 \cdot 3 - 2 = 13, \ldots, \ 5 \cdot 13 - 2 = 63
\]
d) \( \frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \ldots + \frac{1}{49.50} = \sum_{i=1}^{49} \frac{1}{i(i+1)} \), clearly

57. \( \Delta x = \frac{11 - 3}{32} = \frac{4}{32} = \frac{1}{8} \). From the definition on page 341, the midpoint Riemann sum is \( \sum_{k=1}^{n} f(x_k^*) \Delta x \) where \( x_k^* = x + (k - \frac{1}{2}) \Delta x \), so in this case, \( \sum_{k=1}^{32} f\left(3 + (k - \frac{1}{2}) \cdot \frac{1}{4}\right) \cdot \frac{1}{4} = \sum_{k=1}^{32} (3 + \frac{1}{4}k - \frac{1}{6})^3 \cdot \frac{1}{4} \)

\[ \frac{1}{4} \cdot \sum_{k=1}^{32} (2.875 + \frac{k}{4})^3 = \frac{1}{4} \left[ \sum_{k=1}^{32} (2.875^3 + 3 \cdot 2.875^2 \frac{k}{4} + 3 \cdot 2.875 \frac{k^2}{16} + \frac{k^3}{64}) \right] \]

\[ = \frac{1}{4} \left[ (\sum_{k=1}^{32} 2.875^3) + \frac{3 \cdot 2.875^2}{4} \sum_{k=1}^{32} k + \frac{3 \cdot 2.875}{16} \sum_{k=1}^{32} k^2 + \frac{1}{64} \sum_{k=1}^{32} k^3 \right] \]

\[ = \frac{1}{4} \left[ 32 \cdot 2.875^3 + \frac{3 \cdot 2.875^2}{4} \cdot \frac{32(33)}{2} + \frac{3 \cdot 2.875}{16} \cdot \frac{32(33)(65)}{6} + \frac{1}{64} \cdot \frac{32^2(33)^2}{4} \right] \]

The last step uses Theorem 5.1. This would be OK as a final answer, but a calculator shows it's equal to 3639.125, which agrees with the answer in the back of the book.

62. The formula \( \sum_{k=1}^{8} f(1.5 + \frac{k}{2}) \cdot \frac{1}{2} \) looks like the Riemann sum \( \sum_{k=1}^{n} f(x_k^*) \Delta x \) with \( n = 8 \), \( \Delta x = \frac{1}{2} \), and \( x_k^* = 1.5 + \frac{k}{2} = 1.5 + k \Delta x \). There are 8 subintervals of length \( \frac{1}{2} \), so the whole interval has length 4. We know \( x_k^* = 1.5 + \frac{k}{2} = 2 \) must be somewhere in the 1st interval. If we say that 2 is the left endpoint of the 1st subinterval, then the whole interval would be \([2,6]\), and the sum would be a left Riemann sum. So: \( \sum_{k=1}^{8} f(1.5 + \frac{k}{2}) \cdot \frac{1}{2} \) is a left Riemann sum for \( f \) on the interval \([2,6]\) with \( n = 8 \).

(\text{It could also be a right Riemann sum on the interval } [1.5,5.5] \text{ or even a midpoint sum on } [1.75, 5.75].)