

This lab is review for tomorrow's test. It will not be collected. Answers will be available before the end of the lab. This lab does not ask for any definitions, but there might be some definitions on the test.

1. Evaluate the following improper integrals. (Be sure to first write the integral as a limit.)

$$\text{a) } \int_0^1 x^{-3/4} dx \quad \text{b) } \int_2^\infty \frac{1}{x(\ln(x))^3} dx \quad \text{c) } \int_{-\infty}^1 e^{3t} dt$$

2. Solve the initial value problem $\frac{4y^3}{x^2} \frac{dy}{dx} = 1$, $y(3) = 1$

3. Find the general solution of the differential equation $\frac{dy}{dt} = ye^t$

4. Verify that if k is a positive constant, then $y(t) = \frac{32}{k}t + \frac{32}{k^2}(e^{-kt} - 1)$ is a solution of the differential equation $y'' = 32 - ky'$

5. Find the limit of each of the following sequences, or explain why the limit does not exist.

$$\text{a) } \left\{ \frac{3n^4 + 1}{5n - n^3} \right\}_{k=1}^\infty \quad \text{b) } \left\{ \frac{(-1)^{n7^n}}{8^{n+1}} \right\}_{n=0}^\infty \quad \text{c) } \left\{ \frac{n}{n+1} + 2^{-n} \right\}_{n=1}^\infty$$

6. Use the Squeeze Theorem to show that the limit of the sequence $\left\{ 3 + \frac{(-1)^n}{3^n} \right\}_{n=1}^\infty$ is 3.

7. Find the sum of the geometric series $\sum_{n=1}^\infty 5^{-n}e^{n+1}$

8. Given that $\sum_{k=1}^\infty \frac{1}{n^2} = \frac{\pi^2}{6}$, compute the sum of the infinite series $\sum_{n=2}^\infty \left(\frac{2^{n-1}}{5^n} + \frac{5}{n^2} \right)$

9. Apply the *Integral Test* to the series $\sum_{n=1}^\infty \frac{2n}{(n^2 + 1)^2}$ (The function involved is decreasing on $[1, \infty)$; you do **not** have to verify this.)

10. Determine whether each of the following sequences converges or diverges. (Explain your answer, and state which test you use.)

$$\text{a) } \sum_{k=0}^\infty \frac{k+1}{9999 \cdot (k+7)} \quad \text{b) } \sum_{n=1}^\infty \frac{1}{n^7} \quad \text{c) } \sum_{n=3}^\infty (-1)^n 2^{-n} \quad \text{d) } \sum_{i=1}^\infty (-1)^n 2^n$$

11. Explain why the differential equation $\frac{dP}{dt} = 0.02 \cdot P$ could be a reasonable model for population growth.

12. A *bounded increasing sequence must converge*. Explain what this means and why it's true.

13. Carefully explain the difference between a *sequence* and a *series*, and then explain the relationship between the two.