1. Suppose that \( f \) is a function that is continuous on \([0, \infty]\) and that \( \lim_{x \to \infty} f(x) = 1 \). Consider the average value of \( f(x) \) on the interval \([0, b]\). What do you expect to happen to this average value as \( b \to \infty \)? That is, what is \( \lim_{b \to \infty} \left( \frac{1}{b} \int_0^b f(x) \, dx \right) \)? Why? Explain your reasoning. Draw a picture.

2. Consider the function \( f(x) = \begin{cases} x, & 0 \leq x < 2 \\ 3, & 2 \leq x \leq 5 \end{cases} \). This function has a jump discontinuity at \( x = 2 \). (Recall that this means the left and right limits, \( \lim_{x \to 2^-} f(x) \) and \( \lim_{x \to 2^+} f(x) \), both exist but are different.) Nevertheless, \( f \) is integrable on the interval \([0, 5]\), and we can define the area function \( A(x) = \int_0^x f(t) \, dt \).

   a) Draw the graph of \( f \) on the interval \([0, 5]\).

   b) Find an explicit formula for the area function \( A(x) = \int_0^x f(t) \, dt \) on the interval \([0, 5]\). It will be similar to the formula for \( f(x) \); that is, it will be a split function. (Show your work!)

   c) The Fundamental Theorem of Calculus implies that \( A(x) \) is differentiable on any interval on which \( f(x) \) is continuous, but it doesn’t say anything about what happens at \( x = 2 \), where \( f(x) \) has a discontinuity. So, what does happen with \( A(x) \) at \( x = 2 \)? Is \( A \) differentiable at \( x = 2 \)? Is \( A \) continuous at \( x = 2 \)? Why? [Look at what happens as you approach \( x = 2 \) from the left and from the right.]

   d) Try to come up with a hypothesis about what happens to any area function \( A(x) = \int_a^x f(t) \, dt \) at a value of \( x \) where \( f \) has a jump discontinuity. State your hypothesis clearly, and try to specify as much as you can about the behavior of \( A \) at the point where \( f \) is discontinuous.

3. Sometimes, an indefinite integral can be found using a substitution that is less than obvious. Compute the following integrals by using the suggested substitution.

   a) \( \int \frac{x^3}{\sqrt{2x^2 + 1}} \, dx \), using \( u = 2x^2 + 1 \). (Hint: Write \( x^2 \) in terms of \( u \).)

   b) \( \int \frac{1}{\sqrt{x}(1 + x)} \, dx \), using \( u = \sqrt{x} \). (Hint: Write \( x \) in terms of \( u \).)

   c) \( \int \sin^3(x) \, dx \), using \( u = \cos(x) \). (Hint: Write \( \sin^2(x) = 1 - \cos^2(x) \).)
4. In each of the following problems, fill in the box with a non-zero function that will make the integral doable. Then find the resulting integral.

a) \[ \int \Box \cdot \sin(x^2 + 3x + 7) \, dx \]

b) \[ \int \sqrt{x} \cdot e^{\Box} \, dx \]

c) \[ \int \sqrt[3]{\Box} \cdot (3e^{3x} + 3e^{-3x}) \, dx \]

If you have extra time in the lab, consider working on this homework, which is due next Tuesday along with the lab:

Section 6.1, # 8, 30, 34, 40, 60