1. In class, we computed $\int e^{ax} \sin(x) \, dx$. We used integration by parts twice and got an equation that we could solve for the desired integral. Apply the same technique to compute

$$\int \sin(\ln(x)) \, dx$$

2. In class, we derived a reduction formula for $\int x^n e^{ax} \, dx$. Apply integration by parts to the integral

$$\int (\ln(x))^n \, dx$$

to get a reduction formula and then use that reduction formula to find

$$\int (\ln(x))^3 \, dx$$

3. Each of the following two problems requires both integration by parts and a substitution. Compute each integral. Start with a substitution. (To avoid confusion, use $z$ or $\theta$, not $u$, as the variable in the substitution.)

   a) $\int \frac{\ln(\ln(x))}{x} \, dx$
   b) $\int \cos(3\sqrt{x}) \, dx$

4. Assume that $a$ and $b$ are constants and that $a \neq b$. Use algebra to verify that

$$\frac{1}{(x+a)(x+b)} = \frac{A}{x+a} - \frac{A}{x+b} \quad \text{where} \quad A = \frac{1}{b-a}$$

and then use this fact to find the following integrals:

   a) $\int \frac{1}{x^2 + 5x + 6} \, dx$
   b) $\int \frac{1}{z^2 - 1} \, dz$
   c) $\int \frac{e^x}{e^{2x} + 2e^x - 8} \, dx$

For integral c), start by applying a substitution!
5. It is possible to use the following identity to compute certain integrals:

\[
\sin(mx) \sin(nx) = \frac{1}{2} \left( \cos((m - n)x) - \cos((m + n)x) \right)
\]

Note that if \( m = n \), this becomes:

\[
\sin^2(nx) = \frac{1}{2} (1 - \cos(2nx))
\]

a) Use the first identity to compute the indefinite integral \( \int \sin(12x) \sin(7x) \, dx \)

b) Use these identities to show that \( \frac{2}{\pi} \int_0^\pi \sin(mx) \sin(nx) \, dx \) is 0 when \( n \neq m \) and that it is 1 when \( n = m \), where \( n \) and \( m \) are positive integers.

c) Suppose that \( f(x) \) is a function that has the form \( f(x) = \sum_{n=1}^{K} a_n \sin(nx) \) for some positive integer \( K \) and some constants \( a_1, a_2, \ldots, a_K \). Suppose that we know that \( f(x) \) can be written in this form, but we don’t know the constants \( a_n \). Use the properties of integrals and the results from part a) to show that \( a_n \) can be computed as \( a_n = \frac{2}{\pi} \int_0^\pi f(x) \sin(nx) \, dx \). This shows that, starting from the function without knowing the values of the constants \( a_n \), we can use integration to discover their values.

(The function \( \sin(nx) \) represents a “pure tone” or a “pure frequency” of \( n \). \( f(x) \) is a combination of these pure tones. This exercise shows how \( f(x) \) can be analyzed to determine how much of each pure frequency it contains. This applies, for example, to a sound that is made up of a finite number of pure tones. This idea is the beginning of digital signal analysis, which is used in computer processing of sound waves and other signals. Extended to infinite sums, it leads to Fourier analysis, one of the core fields of applied mathematics.)