Math 135, Fall 2019, Homework 10

This homework is due next Wednesday, November 20.

1. Let $A = \{a, b, c, d, e\}$ and let $B = \{1, 2, 3, 4\}$ and let $C = \{R, G, B\}$. Let $f\colon A \to B$ and $g\colon B \to C$ be functions that are defined as sets as follows:

$$f = \{(a, 3), (b, 1), (c, 1), (d, 2), (e, 4)\}$$
$$g = \{(1, R), (2, B), (3, R), (4, G)\}$$

Write out the set that represents the function $g \circ f$.

2. Let $A = \{a, b, c, d, e, f\}$, and let $h\colon A$ be the function defined as a set by

$$f = \{(a, b), (b, c), (c, e), (d, e), (e, f), (f, f)\}$$

a) Write out the set that represents the function $h \circ h$.

b) Write out the set that represents the function $h \circ h \circ h$.

c) Write out the set that represents the function $h \circ h \circ h \circ h$.

d) What happens if you continue to take compositions of h with itself?

3. (*Exercise 12.2.4.*) Let $f\colon \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$ be the function defined by $f(n) = (2n, n+3)$. Is f injective? Is it surjective? Prove your answers.

4. (*Exercise 12.2.14.*) Let A be a set, and let $\theta\colon \mathscr{P}(A) \to \mathscr{P}(A)$ be the function defined by $\theta(X) = \overline{X}$. (Here, $\overline{X}$ is the complement of the set X in A.) Is θ injective? Is it surjective? Is it bijective? Explain. (Hint: This is easy! Use the fact that $\overline{\overline{X}} = X$.)

5. (*Exercise 12.2.18.*) Let $f\colon \mathbb{N} \to \mathbb{Z}$ be the function $f(n) = \frac{1}{4}\big((-1)^n(2n-1)+1\big)$. Prove that f is bijective.

6. Suppose that $f\colon A \to B$ and $g\colon B \to C$ are functions.

a) Show that if $g \circ f$ is surjective, then g is surjective.

b) Give an example to show that when $g \circ f$ is surjective, f is not necessarily surjective.

7. Suppose that $f\colon A \to B$ and $g\colon B \to C$ are functions.

a) Show that if $g \circ f$ is injective, then f is injective.

b) Give an example to show that when $g \circ f$ is injective, g is not necessarily injective.

8. (*Exercise 12.5,2.*) Define $f\colon \mathbb{R} \smallsetminus \{2\} \to \mathbb{R} \smallsetminus \{5\}$ by $f(x) = \frac{5x+1}{x-2}$. The function f is bijective. Find its inverse. (Note: You are **not** asked to show that f is in fact bijective; just assume that it is.)

9. Define $s\colon Z \to Z$ by $s(n) = \begin{cases} n+1 & \text{if } n \text{ is even} \\ n-1 & \text{if } n \text{ is odd} \end{cases}$. The function s is bijective. Show that s is its own inverse function. Can you find any function from $\mathbb{Z}$ to $\mathbb{Z}$ that is its own inverse, besides s and the identity function? (There is one with a very simple formula.)