Math 135, Fall 2019, Homework 3 Answers

2.1.2 This is a statement. It is true because integers are real numbers.

2.1.4 This is not a statement. It's a noun. It doesn't say anything about $\mathbb{Z}$ and $\mathbb{N}$. A related statement would be "$\mathbb{Z}$ and $\mathbb{N}$ are sets."

2.1.8 This is a statement. It is false because $\mathscr{P}(\mathbb{N})$ is the power set of $\mathbb{N}$. Every subset of $\mathbb{N}$ is an element of $\mathscr{P}(\mathbb{N})$, and $\mathbb{N}$ is a subset of $\mathbb{N}$.

2.1.10 This is a statement. It is true: $\mathbb{R} \times \mathbb{N}$ contains ordered pairs (a, n), where the first coordinate is any real number and the second coordinate is a natural number. And similarly, $\mathbb{N} \times \mathbb{R}$ contains ordered pairs (n, a), where the first coordinate is in $\mathbb{N}$and the second coordinate is in $\mathbb{R}$. To be in the intersection, an ordered pair (x, y) must have both first and second coordinate in $\mathbb{N}$. That is, (x, y) must be in $\mathbb{N} \times \mathbb{N}$.

2.3.2 If a function is differentiable, then it is continuous.

2.3.4 If a function is a polynomial, then it is rational.

2.3.10 If the discriminant is negative, then the quadratic equation has no real roots.

2.4.2 A functions has a constant derivative if and only if it is linear.

2.4.4 $a \in \mathbb{Q}$ if and only if $5a \in \mathbb{Q}$.

2.6.2 The fact that the columns for $P \vee (Q \wedge R)$ and $(P \vee Q) \wedge (P \vee R)$ are identical proves that the two expressions are logically equivalent:

P	Q	R	$Q \wedge R$	$P \vee (Q \wedge R)$	$P \vee Q$	$P \vee R$	$(P \vee Q) \wedge (P \vee R)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F
F	F	T	F	F	F	T	F
F	F	F	F	F	F	F	F

2.6.8 The fact that the columns for $(\sim P) \Leftrightarrow Q$ and $(P \Rightarrow (\sim Q)) \wedge ((\sim Q) \Rightarrow P)$ are identical proves that the two expressions are logically equivalent:

P	Q	$\sim P$	$\sim Q$	$(\sim P) \Leftrightarrow Q$	$P \Rightarrow (\sim Q)$	$(\sim Q) \Rightarrow P$	$(P \Rightarrow (\sim Q)) \wedge ((\sim Q) \Rightarrow P)$
T	T	F	F	F	F	T	F
T	F	F	T	T	T	T	T
F	T	T	F	T	T	T	T
F	F	T	T	F	T	F	F

2.6.10 The fact that the columns for $(P \Rightarrow Q) \vee R$ and $\sim((P \wedge \sim Q) \wedge (\sim R))$ are identical proves that the two expressions **are** logically equivalent:

P	Q	R	$P \Rightarrow Q$	$(P \Rightarrow Q) \vee R$	$(P \wedge \sim Q) \wedge (\sim R)$	$\sim((P \wedge \sim Q) \wedge (\sim R))$
T	T	T	T	T	F	T
T	T	F	T	T	F	T
T	F	T	F	T	F	T
T	F	F	F	F	T	F
F	T	T	T	T	F	T
F	T	F	T	T	F	T
F	F	T	T	T	F	T
F	F	F	T	T	F	T

To show another way of doing this kind of problem, I will use the laws of logic to show that the second expression is equivalent to the first: $\sim((P \wedge \sim Q) \wedge (\sim R)) \equiv (\sim(P \wedge \sim Q)) \vee \sim(\sim R) \equiv ((\sim P) \vee \sim(\sim Q)) \vee R \equiv ((\sim P) \vee Q) \vee R \equiv (P \Rightarrow Q) \vee R$

2.6.12 The fact that the columns for $\sim(P \Rightarrow Q)$ and $P \wedge \sim Q$ are identical proves that the two expressions **are** logically equivalent:

P	Q	$P \Rightarrow \sim Q$	$\sim(P \Rightarrow Q)$	$\sim Q$	$P \wedge (\sim Q)$
T	T	T	F	F	F
T	F	F	T	T	T
F	T	T	F	F	F
F	F	T	F	T	F

2.6.14 The fact that the columns for $P \wedge (Q \vee \sim Q)$ and $(\sim P) \Rightarrow (Q \wedge \sim Q)$ are identical proves that the two expressions **are** logically equivalent:

P	Q	$\sim Q$	$Q \vee \sim Q$	$P \wedge (Q \vee \sim Q)$	$\sim P$	$Q \wedge \sim Q$	$(\sim P) \Rightarrow (Q \wedge \sim Q)$
T	T	F	T	T	F	F	T
T	F	T	T	T	F	F	T
F	T	F	T	F	T	F	F
F	F	T	T	F	T	F	F

2.7.2 For every real number x, there is a natural number n such that $x^n \geq 0$. This is true since $x^2 \geq 0$ for any real number x. That is, we can take $n = 2$ in all cases.

2.7.4 Any element of $\mathscr{P}(\mathbb{N})$ is a subset of $\mathbb{R}$. Or, any subset of $\mathbb{N}$ is a subset of $\mathbb{R}$. This is true since $\mathbb{N}$ is a subset of $\mathbb{R}$; that is, every natural number is a real number. So it's true that any set of natural numbers is a set of real numbers.

2.7.6 There is a natural number n such that any element of $\mathscr{P}(X)$ has cardinality less than n. Or, for some natural number n, all subsets of $\mathbb{N}$ have cardinality less than n. This is false since there are subsets of $\mathbb{N}$ with arbitrarily large cardinality.