

Math 135, Fall 2019, Homework 8

*This homework is due next Wednesday, October 30.
Reminder: There is a test next Friday, November 1.*

The first part of the homework is the following exercises from Chapter 10 in the textbook, pages 195–197: Problems number 2, 8, 10, 18, and 34.

A few additional problems:

1. a) Use mathematical induction to prove $\sum_{i=1}^n \frac{1}{i(i+1)} = \frac{n}{n+1}$ for all $n \in \mathbb{N}$.

b) Show that $\frac{1}{i(i+1)} = \frac{1}{i} - \frac{1}{i+1}$. Prove $\sum_{i=1}^n \frac{1}{i(i+1)} = \frac{n}{n+1}$ without using induction, by writing out the terms of the sum, applying this formula, and noting that most of the terms cancel.

2. a) Let r be any number. Use induction to prove that $\sum_{i=0}^n r^i = \frac{1-r^{n+1}}{1-r}$ for all integers $n \geq 0$.

b) Now, prove the same formula without using induction, as follows. Let S be the value of the sum $\sum_{i=0}^n r^i$. Write out a formula for $S - rS$, without using summation notation, and note that many of the terms cancel. Finish the proof by noting $S - rS = (1-r)S$ and solving for S .