Math 135, Fall 2019, Test 2 Information

The second in-class test will take place on Friday, November 1. It will cover Chapters 4 through 10 in the textbook. Of course, you will also need to remember material about logic and sets from Chapters 1 and 2. Chapters 4 to 10 concentrate on general proof techniques, but they also include some new mathematical terms and concepts. As usual, the test will include definitions and other essay-type questions, as well as problems testing mathematical concepts such as divisibility, congruence modulo n, and rational and irrational numbers. There will also be several proofs, using different proof techniques, including at least one proof by induction. Given the time constraints of a one-hour test, the proofs should be fairly straightforward.

Proof techniques that you should know:

direct proof of a “P ⇒ Q” statement
proving a “for all” statement
proof by contrapositive
proof by contradiction
proof by cases
if-and-only-if proof
existence proof
proving \( a \in A, A \subseteq B, \) and \( A = B \) for sets \( A \) and \( B \)
disproof by counterexample
proof by mathematical induction
proof by strong induction

Other terms and ideas that you should know for the test:

theorems, lemmas, corollaries
conjectures
even and odd numbers
divisibility for integers, \( a \mid b \) if and only if \( b = ka \) for some \( k \in \mathbb{Z} \)
prime numbers
there are infinitely many prime numbers
greatest common divisor, \( \text{gcd}(a, b) \)
relatively prime integers, \( \text{gcd}(a, b) = 1 \)
congruence modulo \( n \): \( a \equiv b \pmod{n} \)
rational number: can be written as \( \frac{p}{q} \) where \( p, q \in \mathbb{Z} \) and \( q \neq 0 \)
irrational number: a real number that is not rational
\( \sqrt{2} \) is irrational; \( \sqrt{p} \) is irrational for any prime number \( p \)
\( \pi \) is irrational
base case of an induction; inductive case of an induction
inductive hypothesis
factorials:
\[
 n! = n \cdot (n - 1) \cdot (n - 2) \cdots 2 \cdot 1; \quad 0! = 0; \quad (n + 1)! = (n + 1) \cdot n! \text{ for } n > 0
\]
summation notation, \( \sum_{i=1}^{n} a_i \)
why proof by contradiction works
why disproof by counterexample works
why induction works; why the base case is necessary
how validity of induction follows from the well-ordering principle for \( \mathbb{N} \)
well-ordering principle for \( \mathbb{N} \):
Every non-empty set of natural numbers has a smallest element
division algorithm:
If \( a \in \mathbb{Z} \) and \( b \in \mathbb{N} \), then \( a = bq + r \) for some unique \( q \in \mathbb{Z} \) and \( 0 \leq r < b \).
if \( p \) is a prime number and \( p \mid ab \), then \( p \mid a \) or \( p \mid b \)
gcd(\( a, b \)) can be written as \( \gcd(a, b) = ax + by \) for some integers \( x \) and \( y \)
every integer is congruent mod \( n \) to exactly one of 0, 1, \ldots, \( n - 1 \)
Fundamental Theorem of Arithmetic:
Every integer \( n \geq 2 \) has a unique prime factorization