This homework is due by 11:59 PM on Tuesday, December 1

Problem 1. A few problems on complex numbers...

(a) Recall that for a complex number $z = a + bi$, $|z|$ is defined to be $|z| = \sqrt{a^2 + b^2}$. Verify that for two complex numbers z and w, $|zw| = |z| \cdot |w|$.

(b) Suppose that $w_0 = a + bi$ is some complex number. Recall that the conjugate of w_0 is defines as $\overline{w_0} = a - bi$. Let $p(z)$ be the polynomial $p(z) = (z - w_0)(z - \overline{w_0})$. Verify that when $p(z)$ is written in standard form as $p(z) = c_0 + c_1 z + c_2 z^2$, all of the coefficients c_0, c_1, and c_2 are real.

(c) Use the identity $e^{i\theta} = \cos(\theta) + i\sin(\theta)$ and the fact that $(e^{i\theta})^2 = e^{2i\theta}$ to derive the usual double angle formulas: $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$ and $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$.

Problem 2. Find all the eigenvalues, real or complex, of the following matrices. (Note that one of these is really easy.)

(a) $\begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$ **(b)** $\begin{pmatrix} 3+i & 0 & 5 \\ 0 & 2 & 7 \\ 0 & 0 & -i \end{pmatrix}$ **(c)** $\begin{pmatrix} 5 & 0 & 0 \\ -2 & 3 & 6 \\ 0 & 1 & -2 \end{pmatrix}$

Problem 3. Suppose that A is an $n \times n$ matrix, and λ is an eigenvalue for A. Show that λ^2 is an eigenvalue for AA. [Hint: Let $\vec{v}$ be an eigenvector for A with eigenvalue λ.]

Problem 4. Let $h\colon \mathbb{C}^2 \to \mathbb{C}^2$ be a homomorphism that has eigenvalues $1 - i$ and $1 + i$. Suppose that $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$ is an eigenvector with eigenvalue $1 - i$, and that $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ is an eigenvector with eigenvalue $1 + i$. Find the matrix for h in the standard basis, $\langle \vec{e}_i, \vec{e}_2 \rangle$. [Hint: You need to find $h(\vec{e}_1)$ and $h(\vec{e}_2)$.]

Problem 5. Let $\mathscr{D}$ be the vector space of differentiable functions from $\mathbb{R}$ to $\mathbb{R}$. That is, $\mathscr{D}$ is the set $\{f\colon \mathbb{R} \to \mathbb{R} \mid f'(x) \text{ exists for all } x\}$, with the usual addition and scalar multiplication for real-valued functions. Let $\partial\colon \mathscr{D} \to \mathscr{D}$ be the derivative function, $\partial(f) = f'$. Show that every real number λ is an eigenvalue for ∂, and find an eigenvector for eigenvalue λ. [Hint: What is the derivative of e^{ax}? Once you remember that derivative, this question is trivial.]