This homework is due by the end of the day on Thursday, Sept. 24.
Don't forget to show your work and justify your answers!

Problem 1. The following questions about spans can be answered by solving linear systems of equations.

(a) Is the vector $\begin{pmatrix}1\\2\\3\end{pmatrix}$ in the span of the set $T = \left\{ \begin{pmatrix}-1\\0\\1\end{pmatrix}, \begin{pmatrix}1\\1\\1\end{pmatrix} \right\}$ in $\mathbb{R}^3$?

(b) Is the vector $\begin{pmatrix}1\\1\\1\end{pmatrix}$ in the span of the set $T = \left\{ \begin{pmatrix}2\\-1\\1\end{pmatrix}, \begin{pmatrix}0\\1\\4\end{pmatrix}, \begin{pmatrix}-2\\2\\3\end{pmatrix} \right\}$ in $\mathbb{R}^3$?

Problem 2. Let S be the subset $T = \left\{ \begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}1\\1\\0\end{pmatrix}, \begin{pmatrix}1\\1\\1\end{pmatrix} \right\}$ of $\mathbb{R}^3$. Show that $[S]$, the span of S, is all of $\mathbb{R}^3$. (This is asking you to show that every $\vec{v} \in \mathbb{R}^3$ can be written as a linear combination of the vectors in S.)

Problem 3. Let $\mathscr{P}$ be the (infinite-dimensional) vector space of all polynomials. Let T be the subset of $\mathscr{P}$ given by $\{p_0(x), p_1(x), p_2(x), \dots\}$, where $p_0(x) = 1$, $p_1(x) = 1 + x$, $p_2(x) = 1+x+x^2$, $p_3(x) = 1+x+x^2+x^3$, and, more generally, $p_n(x) = 1+x+x^2+\cdots+x^n$. Show that $[T]$, the span of T, is all of $\mathscr{P}$. (You just need to check that any polynomial $q(x) = a_0 + a_1x + a_2x^2 + \cdots + a_kx^k$ can be written as a linear combination of some finite number of elements of T. Hint: Problems 2 and 3 are almost the same question.)

Problem 4. Show that the following vectors in $\mathbb{R}^3$ are linearly dependent. (Recall that vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3$ are linearly dependent if $a\vec{v}_1 + b\vec{v}_2 + c\vec{v}_3 = 0$ for some $a, b, c \in \mathbb{R}$ where a, b, and c are not all zero.)

$$\begin{pmatrix}2\\-1\\1\end{pmatrix}, \begin{pmatrix}0\\1\\4\end{pmatrix}, \begin{pmatrix}-2\\2\\3\end{pmatrix}$$

Problem 5. Let $\mathscr{P}_3$ be the vector space of all polynomials of degree less than or equal to 3. Show that the following vectors in $\mathscr{P}_3$ are linearly independent: $1 - 2x$, $3x - 2x^2 + x^3$, $4 + x^2$.