Problem 1. Let \((M, d)\) be a metric space, and let \(A\) be a subset of \(M\). Prove that \(A\) is open if and only if \(A\) is equal to a union of open balls.

Problem 2. Consider the metric space \((\mathbb{R}, d)\), where \(d\) is the usual metric on \(\mathbb{R}\). For each \(n = 1, 2, 3, \ldots\), let \(O_n\) be the open set \(O_n = (1, 1 + \frac{1}{n})\). Show that \(\{O_n \mid n = 1, 2, \ldots\}\) is an infinite collection of open sets whose intersection is not open. And find an infinite collection of closed subsets of \((\mathbb{R}, d)\) whose union is not closed.

Problem 3. Let \(X\) be any non-empty, bounded subset of \(\mathbb{R}\), and let \(\lambda\) be the least upper bound of \(X\). Show that \(\lambda \in \overline{X}\). That is, the least upper bound of any set is an element of the closure of that set. [Hint: Use the definition of closure of \(X\) as the set of all points of \(X\) plus all accumulation points of \(X\), and use Problem 1 from Homework 3.]

Problem 4. Let \((A, \sigma)\), \((B, \tau)\), and \((C, \eta)\) be metric spaces. Let \(f: A \to B\) and \(g: B \to C\). Suppose that \(f\) and \(g\) are continuous functions. Prove that their composition, \(g \circ f\), is a continuous function.

Problem 5. Let \(\{x_n\}_{n=1}^\infty\) be a convergent sequence in a metric space. Show that its limit is unique. That is, prove the following statement: if \(\{x_n\}_{n=1}^\infty\) converges to \(y\) and \(\{x_n\}_{n=1}^\infty\) converges to \(z\), then \(y = z\).