Induction

MATH 135: First Steps into Advanced Mathematics Due: April 1, 2011

Name (Print):

Suppose you were to prove each of the following theorems using induction. What would your P_k statement be? (That is, for each of the following determine P_k . Do **not** actually try to prove the statement. Remember that P_k should be a sentence. You do not have to understand all the parts of the statement in order to find P_k .)

1. For every
$$n \in \mathbb{N}$$
, $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$.

2. In a line of at least two people, if the first person is a woman and the last person is a man, then somewhere in the line there is a man standing immediately behind a woman.

3. For every integer
$$n \ge 0$$
, $\sum_{i=0}^{n} 2^{i} = 2^{n+1} - 1$.

4. If x > 1 is a given real number, then for every natural number $n \ge 2$, $(1 + x)^n > 1 + nx$.

5. A planar graph on n vertices can be colored with 5 colors.

6. For all natural numbers a and b, there exists a natural number s such that a < sb. (Do induction on either a or b.)

7. Every natural number n > 1 has a prime factor.

8. Choose n + 2 distinct points from the circumference of a circle. If consecutive points along the circle are joined by line segments creating a polygon with n + 2 sides, then the sum of the interior angles of the resulting polygon equals 180n degrees.

9. For every natural number $n, 1 + 3 + 5 + ... + (2n - 1) = n^2$.

10. For every natural number $n, n^2 - n$ is even.