LAB 1

MATH 130 Section 2
January 24, 2019
Covering Precalculus

Your Name (Print): ANSWER KEY

This lab contains some exercises that represent situations that show up in the middle of calculus problems. Some ask you to solve for a variable, and some ask you merely to simplify an expression. Simplifying and canceling can be incredibly helpful in making a complex problem more tractable. You should look for opportunities to do so, and always check your work!

1. Simplify each of these expressions writing a single power of x.

(a) $\frac{\sqrt{x^8}}{2x^2} = \frac{x^4}{2x^2}$

$= \frac{x^2}{2} = \frac{1}{2}x^2$

(b) $\frac{x^3x^{-5}}{2x^6x^{-1}} = \frac{x^{3-5}}{2x^{6-1}} = \frac{x^{-2}}{2x^5} = \frac{1}{2x^5x^2} = \frac{1}{2x^{5+2}} = \frac{1}{2}\frac{1}{x^7} = \frac{1}{2}x^{-7}$

(c) $\left(\frac{x^{3/5}}{x^{-2}}\right)^4 = \frac{x^{3/5 \cdot 4}}{x^{-2 \cdot 4}} = \frac{x^{\frac{12}{5}}}{x^{-8}} = x^{12/5} \cdot x^8 = x^{12/5 + \frac{40}{5}} = x^{52/5}$

2. (a) What is the definition of a rational function?

A rational function is a function $R(x)$ that can be expressed as a ratio of two polynomials, $P(x)$ and $Q(x)$, so that $R(x) = \frac{P(x)}{Q(x)}$.

(b) Simplify this rational function: $\frac{8(x+1)^4 + 7x^3(x+1)^2}{(x+1)^4}$.

$= \frac{(x+1)^2\left[8(x+1)^2 + 7x^3\right]}{(x+1)^2(x+1)^2} = \frac{8(x^2+2x+1) + 7x^3}{(x+1)^2}$

$= \frac{7x^3 + 8x^2 + 16x + 8}{(x+1)^2}$

(c) Simplify this rational function: $\dfrac{9(x-2)^3(x+1)-2(x-2)^4(x-1)^2}{(x-2)^6}$.

$$= \frac{\cancel{(x-2)^3}\left[9(x+1)-2(x-2)(x-1)^2\right]}{\cancel{(x-2)^3}(x-2)^3}$$

$$= \frac{9(x+1)-2(x-2)(x-1)^2}{(x-2)^3}$$

3. Find the domain of each function and express your answers in interval notation. Be sure to explain your reasoning.

(a) $f(x) = \dfrac{7x+2}{4x-9}$

Eliminate zeros in the denominator:

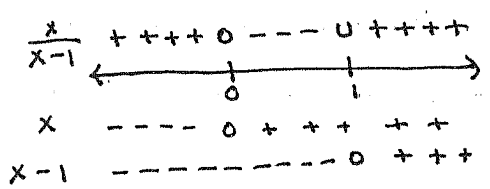
$$4x-9 \neq 0 \Rightarrow 4x \neq 9 \Rightarrow x \neq \frac{9}{4}$$

Domain: $\left(-\infty, \frac{9}{4}\right) \cup \left(\frac{9}{4}, \infty\right)$

(b) $h(x) = \sqrt{\dfrac{x}{x-1}}$

Eliminate zeros in the denominator: $x-1 \neq 0 \Rightarrow x \neq 1$

Eliminate negatives under the square root: $\dfrac{x}{x-1} \geq 0$



Domain: $(-\infty, 0] \cup (1, \infty)$

4. Find **ALL** solutions of $|5x-3| = 13$.

$5x - 3 = 13$ or $5x - 3 = -13$

$5x = 16$ $\qquad$ $5x = -10$

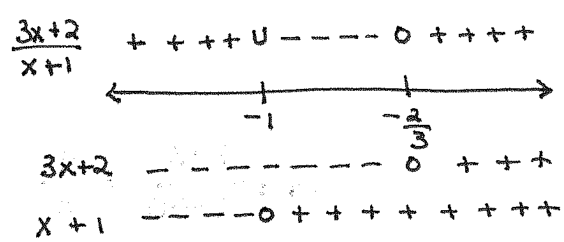
$x = \frac{16}{5}$ or $x = -2$

5. Give in interval notation in the simplest possible form the set of x for which $\frac{4x+3}{(x+1)} \leq 1$. (Hint: When solving inequalities and equations with more than one term, begin by rewriting it with zero on one side.)

$$\frac{4x+3}{x+1} - 1 \leq 0$$

$$\frac{4x+3-(x+1)}{x+1} \leq 0$$

$$\frac{3x+2}{x+1} \leq 0$$



Solution: $x \in \left(-1, -\frac{2}{3}\right]$

6. Simplify $\frac{z^{-1}+w^{-1}}{(zw)^{-1}}$ so that there are no fractions or negative powers in either the numerator or the denominator.

$$\frac{z^{-1}+w^{-1}}{(zw)^{-1}} = \frac{\frac{1}{z}+\frac{1}{w}}{\frac{1}{zw}} = \left(\frac{1}{z}+\frac{1}{w}\right)\cdot zw = w+z$$

7. Simplify $\dfrac{\cos^2(x)+\sin^2(x)}{\cos^2(x)-\sin^2(x)}$ as much as possible.

$$= \frac{1}{\cos 2x}$$

$$= \sec 2x$$

8. You will very soon need to use much of the information you already know about lines, their slopes and their equations. Here are several problems that you should be able to do quickly. If you are feeling rusty on terminology, check out Appendix A.

(a) Find an equation of the line through the point $(3,4)$ that has slope $-\dfrac{1}{2}$.

$$y-4 = -\tfrac{1}{2}(x-3)$$

$$y = -\tfrac{1}{2}x + \tfrac{3}{2} + 4$$

$$y = -\tfrac{1}{2}x + \tfrac{11}{2}$$

(b) Find an equation of the line that passes through the points $(-1,3)$ and $(2,7)$.

$$m = \frac{7-3}{2-(-1)} = \frac{4}{3}$$

$$y-7 = \tfrac{4}{3}(x-2)$$

$$y = \tfrac{4}{3}x - \tfrac{8}{3} + 7$$

$$y = \tfrac{4}{3}x + \tfrac{13}{3}$$

(c) Find the slope of the line $3x-2y=7$.

$$-2y = -3x + 7$$

$$y = \tfrac{3}{2}x - \tfrac{7}{2}$$

$$\Rightarrow m = \tfrac{3}{2}$$

(d) Find an equation of the line parallel to $3x-2y=7$ and passing through the point $(-2,2)$. Leave your final answer in slope-intercept form.

from (c), $m = \tfrac{3}{2}$

$$y-2 = \tfrac{3}{2}(x+2)$$

$$y = \tfrac{3}{2}x + 3 + 2$$

$$y = \tfrac{3}{2}x + 5$$

9. Suppose $g(x) = \frac{2}{x-1}$ and $h \neq 0$. Find the following and simplify your solution as much as possible. (Leaving it in factored form is fine.)

(a) $g(-3)$

$$g(-3) = \frac{2}{-3-1} = \frac{2}{-4} = -\frac{1}{2}$$

(b) $g(x+h)$

$$g(x+h) = \frac{2}{(x+h)-1} \quad \text{or} \quad \frac{2}{x+h-1}$$

(c) $\frac{g(x+h)-g(x)}{h}$

$$= \frac{\frac{2}{x+h-1} - \frac{2}{x-1}}{h} = \frac{\frac{2(x-1)-2(x+h-1)}{(x+h-1)(x-1)}}{h}$$

$$= \frac{2x-2-2x-2h+2}{h(x-1)(x+h-1)}$$

$$= \frac{-2h}{h(x-1)(x+h-1)} = \frac{-2}{(x-1)(x+h-1)}$$

10. Factor the following polynomials:

(a) $20z^2 - 17z + 3$

$$(5z-3)(4z-1)$$

(b) $4x^7 - 64x^3$

$$= 4x^3(x^4-16)$$

$$= 4x^3(x^2+4)(x^2-4)$$

$$= 4x^3(x^2+4)(x+2)(x-2)$$