WEEK 1 LAB

MATH 131: Calculus II
January 23, 2020
Reviewing Calculus I

Your Name (Print): ANSWER KEY

1. Evaluate the following integrals. Don't forget your family!

(a) $\int x^3(x^7+4x^2+8)dx$

$$= \int (x^{10} + 4x^5 + 8x^3)dx$$

$$= \frac{1}{11}x^{11} + \frac{4}{6}x^6 + \frac{8}{4}x^4 + C$$

$$= \frac{1}{11}x^{11} + \frac{2}{3}x^6 + 2x^4 + C$$

(b) $\int \frac{x^3\cos x + x\sec x + \cos x}{x\cos x}dx$

$$= \int \left[\frac{x^3\cos x}{x\cos x} + \frac{x\sec x}{x\cos x} + \frac{\cos x}{x\cos x}\right]dx$$

$$= \int \left(x^2 + \sec^2 x + \frac{1}{x}\right)dx$$

$$= \frac{x^3}{3} + \tan x + \ln|x| + C$$

(c) $\int \frac{\sin x}{1-\sin^2 x}dx$

$$= \int \frac{\sin x}{\cos^2 x}dx$$

$$= \int \left(\frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}\right)dx$$

$$= \int \tan x \sec x\, dx = \sec x + C$$

2. Solve the differential equation $f''(x) = 120x^3 - 42x$, $f(-1) = -5$, $f(2) = 136$.

$$f'(x) = \int f''(x)\,dx = \int (120x^3 - 42x)\,dx$$

$$= \frac{120}{4}x^4 - \frac{42}{2}x^2 + C = 30x^4 - 21x^2 + C$$

$$f(x) = \int f'(x)\,dx = \int (30x^4 - 21x^2 + C)\,dx$$

$$= \frac{30}{5}x^5 - \frac{21}{3}x^3 + Cx + D$$

$$= 6x^5 - 7x^3 + Cx + D$$

+5

$$f(-1) = -6 + 7 - C + D = 1 - C + D = -5.$$

$$D - C = -6$$

$$f(2) = 6(32) - 7(8) + 2C + D = 192 - 56 + 2C + D = 136$$

$$136 = 2C + D + 136$$

$$2C + D = 0$$

$$2(-C + D = -6)$$

$$3D = -12$$

So $D = -4$ and $(-4) - C = -6$ or

$-C = -2 \Rightarrow C = 2$

Thus $f(x) = 6x^5 - 7x^3 + 2x - 4$.

3. Evaluate the following using the theorem on the Calculus I Review handout, and **explain your answers**. Be careful!

(a) $\displaystyle\lim_{x\to\infty} \frac{8x^4 + 5x - 12}{3 + 7x + 9x^4} = \frac{8}{9}$

since $k = n = 4$ and $\frac{a_k}{b_n} = \frac{8}{9}$

+2

(b) $\displaystyle\lim_{x\to\infty} \frac{5x^6 + 3x - 9}{-2x^5 + 6x^3 - 10x + 13} = -\infty$

since $k = 6$, $n = 5$ and $k > n$, also $\frac{a_k}{b_n} = -\frac{5}{2}$ is negative

+2

4. (a) State the definition of an antiderivative. (You can look this up in your textbook if you don't remember the official definition.)

The function $F(x)$ is an antiderivative of the function $f(x)$ if $F'(x) = f(x)$.

(b) Find <u>an</u> antiderivative of $f(x) = e^x - \cos x + 3$.

$$F(x) = e^x - \sin x + 3x$$

(c) Use your definition in part (a) to justify that your answer in part (b) is correct.

$F'(x) = e^x - \cos x + 3 = f(x)$, therefore

$F(x)$ is an antiderivative of $f(x)$.

(d) Find <u>an</u> antiderivative of $g(x) = \dfrac{1}{1+x^2} + \sec^2 x$.

$$G(x) = \arctan x + \tan x + 7$$

(e) Use your definition in part (a) to justify that your answer in part (d) is correct.

$G'(x) = \dfrac{1}{1+x^2} + \sec^2 x = g(x)$, therefore

$G(x)$ is an antiderivative of $g(x)$.

5. A sprinter in a 100m race explodes off the starting block with an acceleration of 5m/s^2 which she sustains for the first 2 seconds. Her acceleration then drops to 0 for the remainder of the race. What is her time for the race? You will have to divide the race into two segments.

First part of race:

$$a(t) = 5\ m/s^2 \qquad v(0) = 0\ m/s \qquad s(0) = 0m$$

$$\Rightarrow v(t) = \int a(t)\,dt = \int 5\,dt = 5t + C$$

$$v(0) = 0 \Rightarrow C = 0 \quad \text{so} \quad v(t) = 5t$$

$$s(t) = \int v(t)\,dt = \int 5t\,dt = \frac{5t^2}{2} + C$$

$$s(0) = 0 \Rightarrow s(t) = \frac{5}{2}t^2$$

Thus $s(2) = \frac{5}{2}(2)^2 = \frac{5 \cdot 4}{2} = 10\,m$

$v(2) = 5(2) = 10\ m/s$

Second part of race:

$v(t) = 10\ m/s$ and there are $90\,m$ left so she will take 9 more seconds.

Total time: $2 + 9 = 11$ seconds.

6. A toy bumper car is moving back and forth along a straight track. Its acceleration is $a(t) = \cos t + \sin t$. Find the particular velocity and position functions given that $v(\pi/4) = 0$ and $s(\pi) = 3$.

$$v(t) = \int a(t)\,dt = \int (\cos t + \sin t)\,dt$$

$$= \sin t - \cos t + C$$

$$v(\pi/4) = 0 \Rightarrow 0 = \sin\frac{\pi}{4} - \cos\frac{\pi}{4} + C$$

$$0 = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} + C \Rightarrow C = 0$$

Thus $s(t) = \int v(t)\,dt = \int (\sin t - \cos t)\,dt$

$$= -\cos t - \sin t + C$$

$$s(\pi) = 3 \Rightarrow 3 = -\cos\pi - \sin\pi + C$$

$$3 = -(-1) - 0 + C \Rightarrow C = 2$$

Hence $s(t) = -\cos t - \sin t + 2$.

7. We will learn a technique of integration called u-substitution at the end of Chapter 5. Right now, however, it is assumed that you do NOT know this method. Without using u-substitution, evaluate the following integrals and then for each one write a short sentence explaining why you know your answer is correct using the definition of antiderivative.

(a) $\int (4x^3 + 7)\cos(x^4 + 7x)dx$

$= \sin(x^4+7x) + C$

since $\frac{d}{dx}\left[\sin(x^4+7x)+C\right] = \cos(x^4+7x) \cdot (4x^3+7) + 0$

(b) $\int (9x^2 - 10x)e^{3x^3-5x^2}dx$

$= e^{3x^3-5x^2} + C$

since $\frac{d}{dx}\left[e^{3x^3-5x^2} + C\right] = e^{3x^3-5x^2} \cdot (9x^2-10x) + 0$

(c) $\int \frac{\sec x \tan x}{\sec x}dx$

$= \ln|\sec x| + C$

since $\frac{d}{dx}\left[\ln|\sec x| + C\right] = \frac{1}{\sec x} \cdot \sec x \tan x + 0$